

GISAXS/GIWAXS Data Analysis: Thinking in Reciprocal-Space



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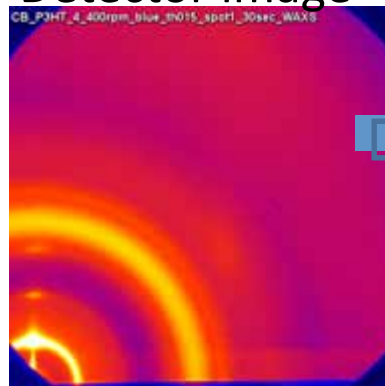


What to do with data?

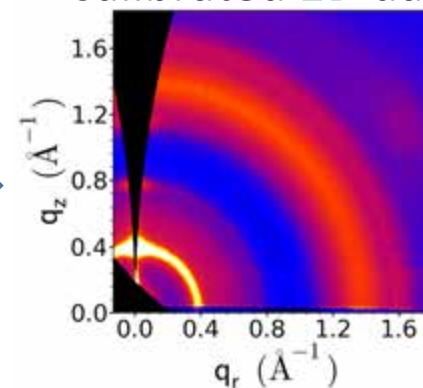
- Qualitative inspection
- Linecuts
- Peak indexing
- Modeling (comparison)
- Fitting

Workflow

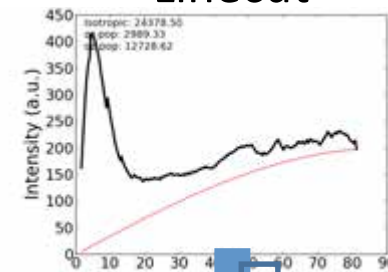
Detector image



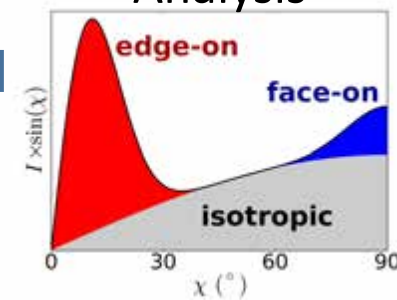
Calibrated 2D data



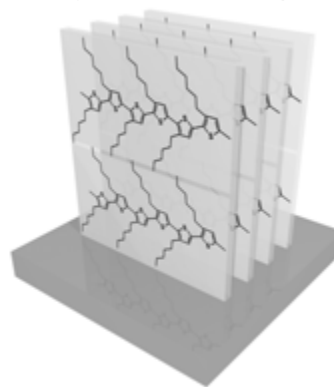
Linecut



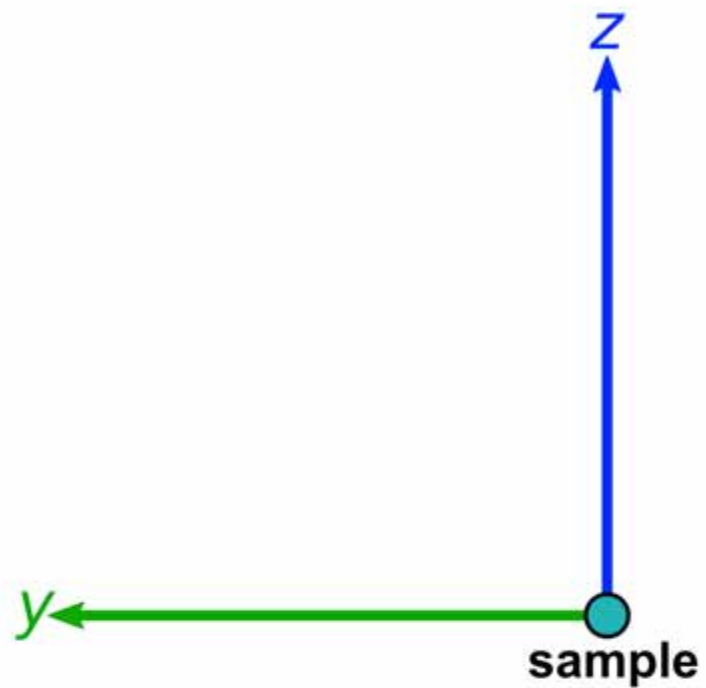
Analysis



Physical insight

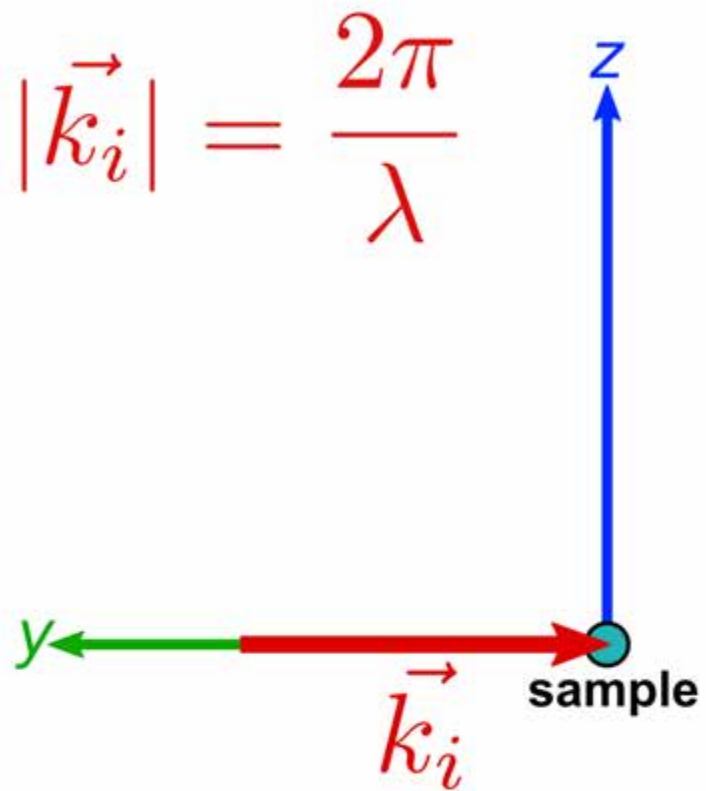


Scattering Geometry



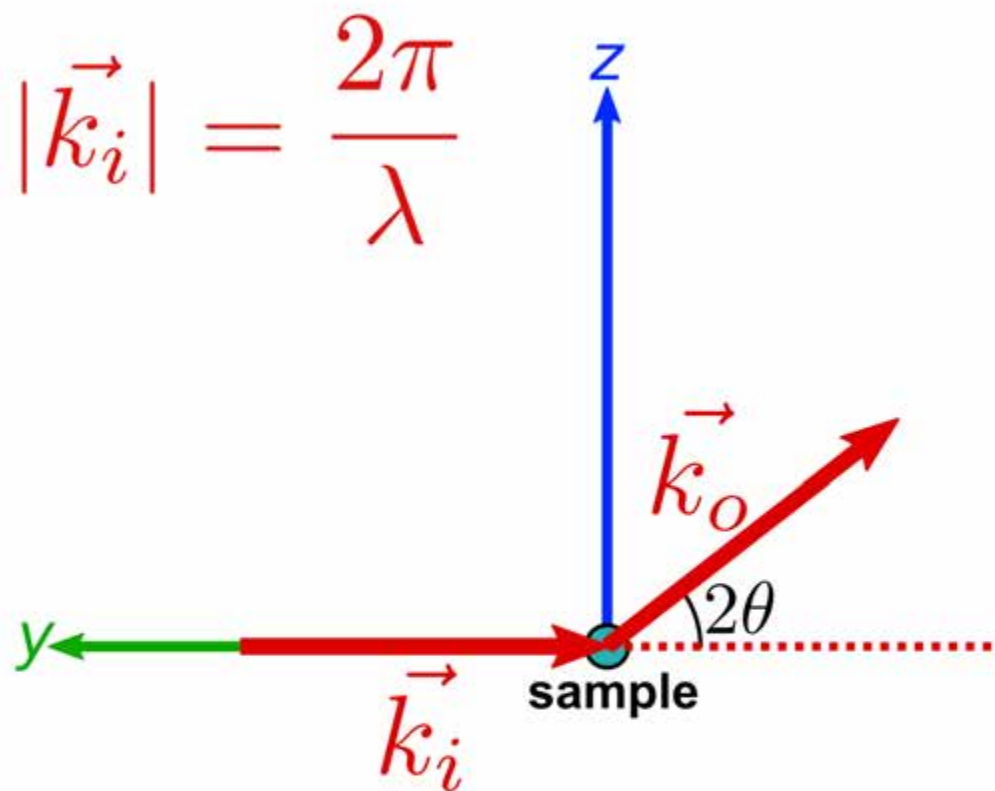
Scattering Geometry

- X-ray beam hits sample



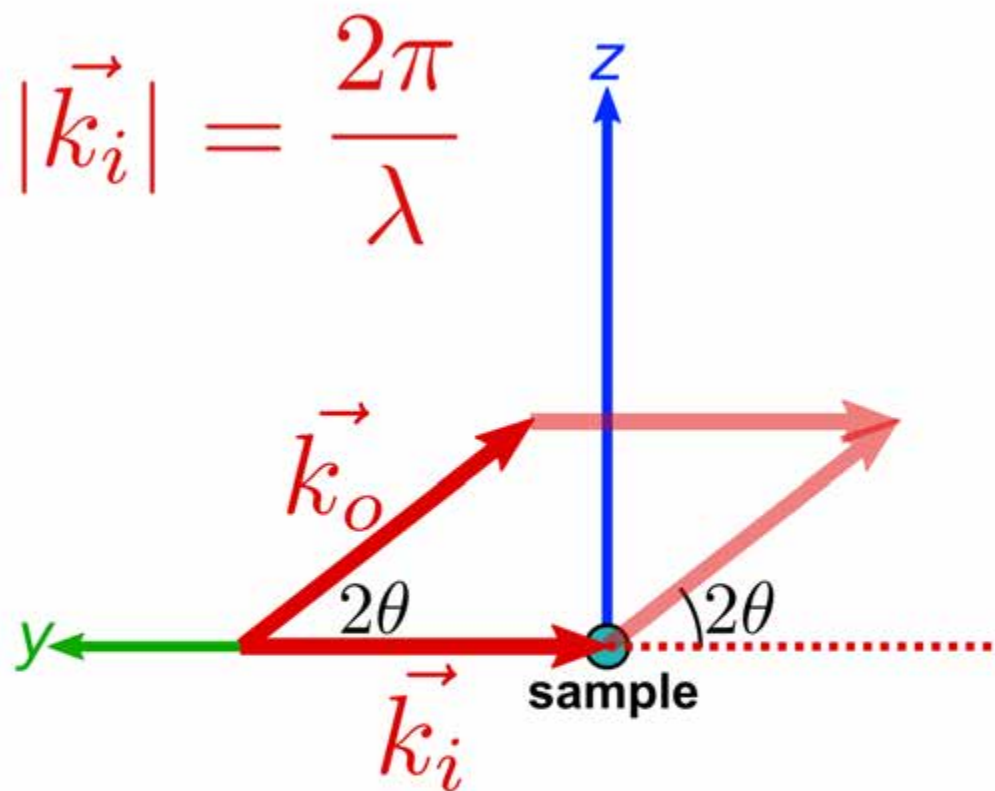
Scattering Geometry

- X-ray beam scatters at a certain angle



Scattering Geometry

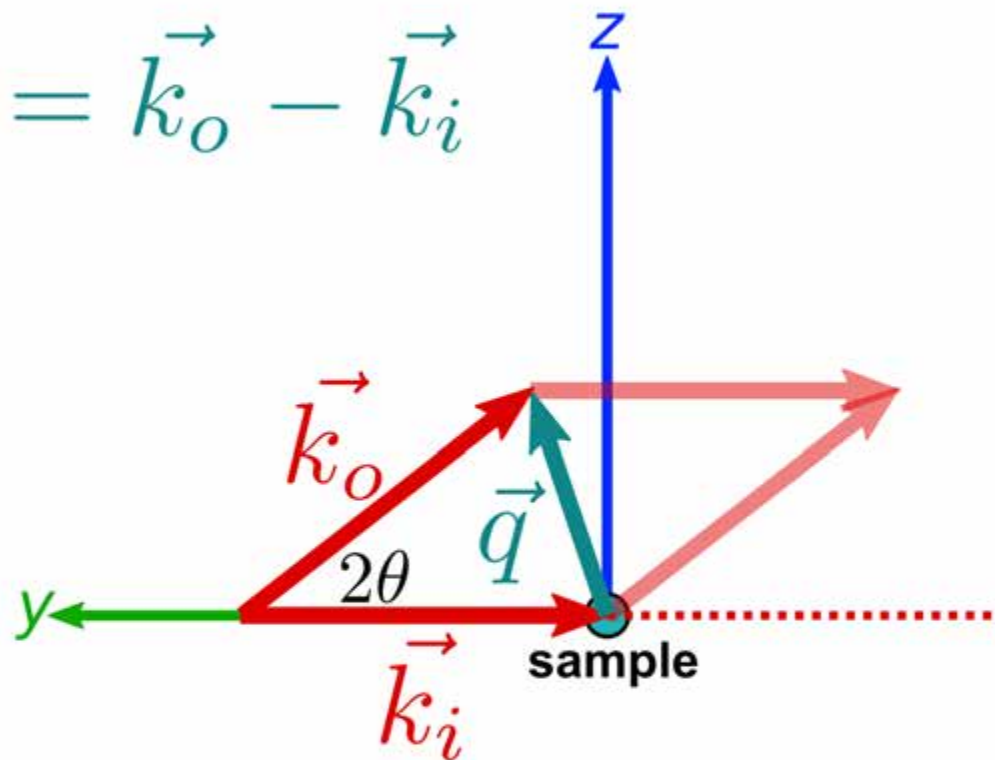
- Consider the difference between the incident and scattered vectors



Scattering Geometry

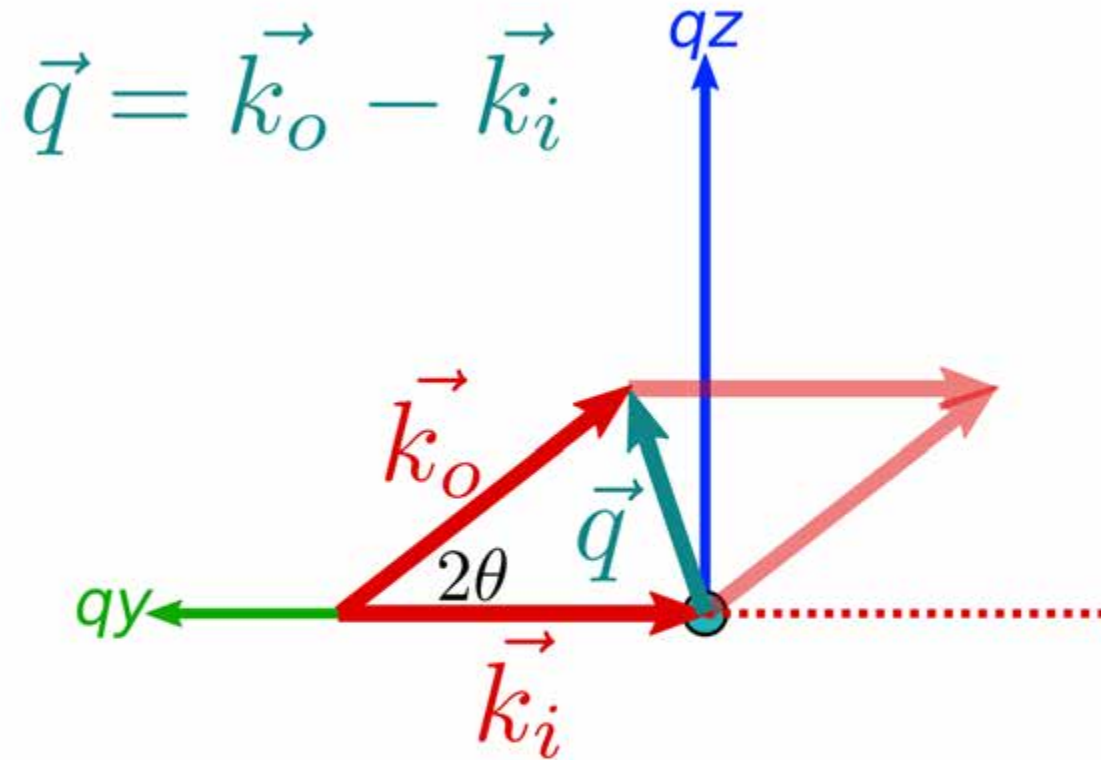
- We define q to be the momentum transfer

$$\vec{q} = \vec{k}_o - \vec{k}_i$$



Reciprocal-space

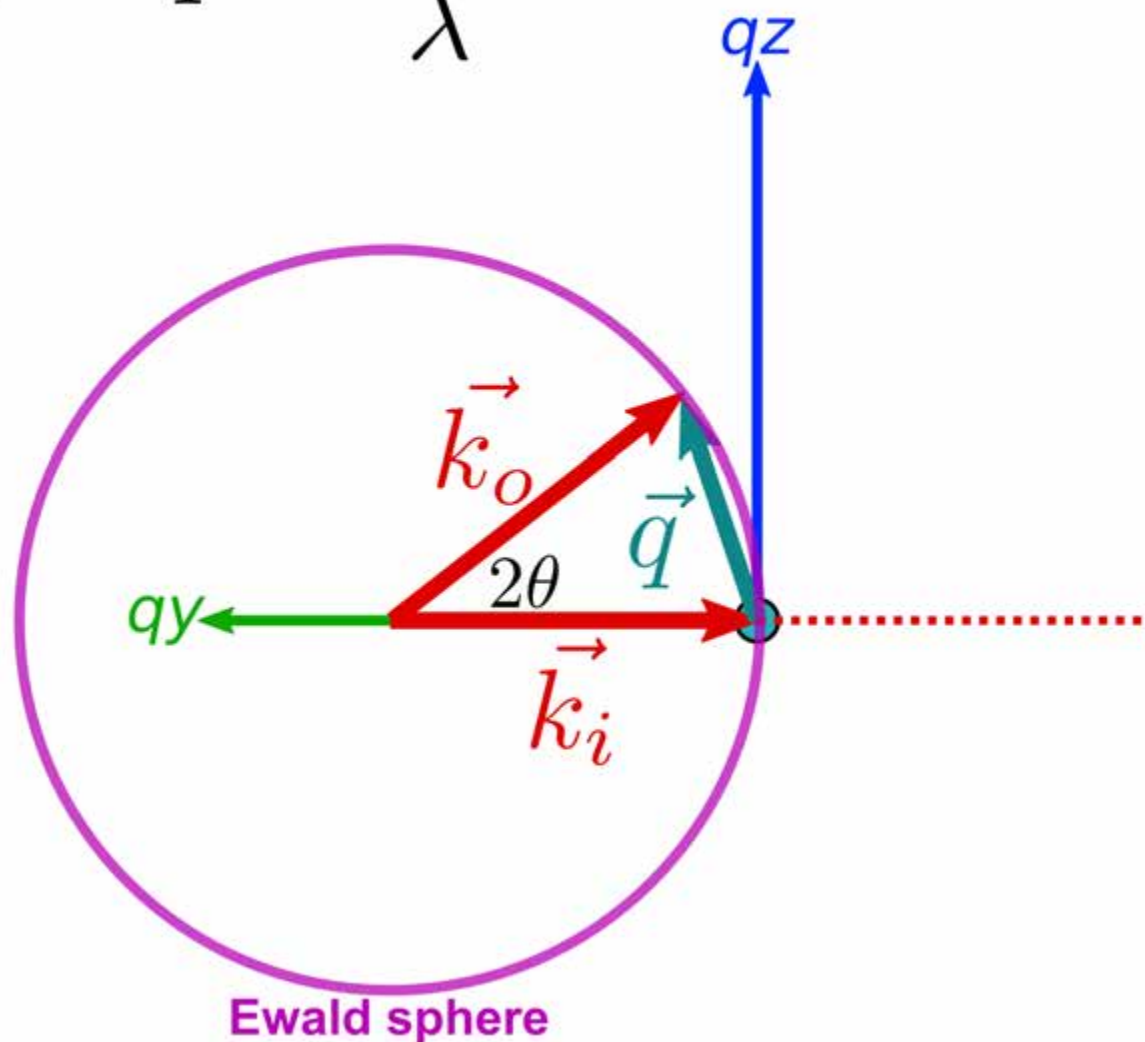
- q is a vector in reciprocal-space (k and q have units of 1/distance)



Reciprocal-space

- q is always on a sphere

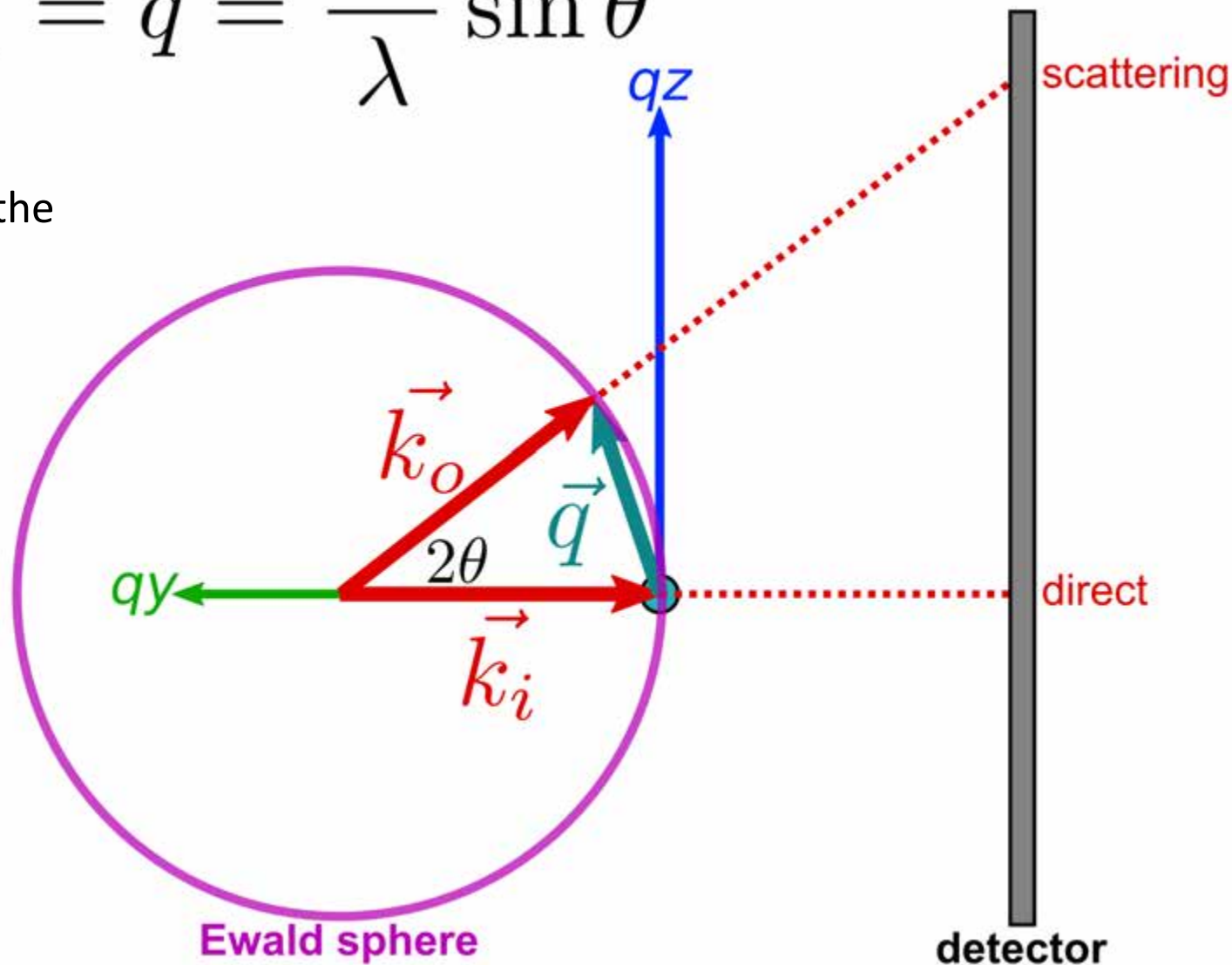
$$|\vec{q}| = q = \frac{4\pi}{\lambda} \sin \theta$$



Reciprocal-space

$$|\vec{q}| = q = \frac{4\pi}{\lambda} \sin \theta$$

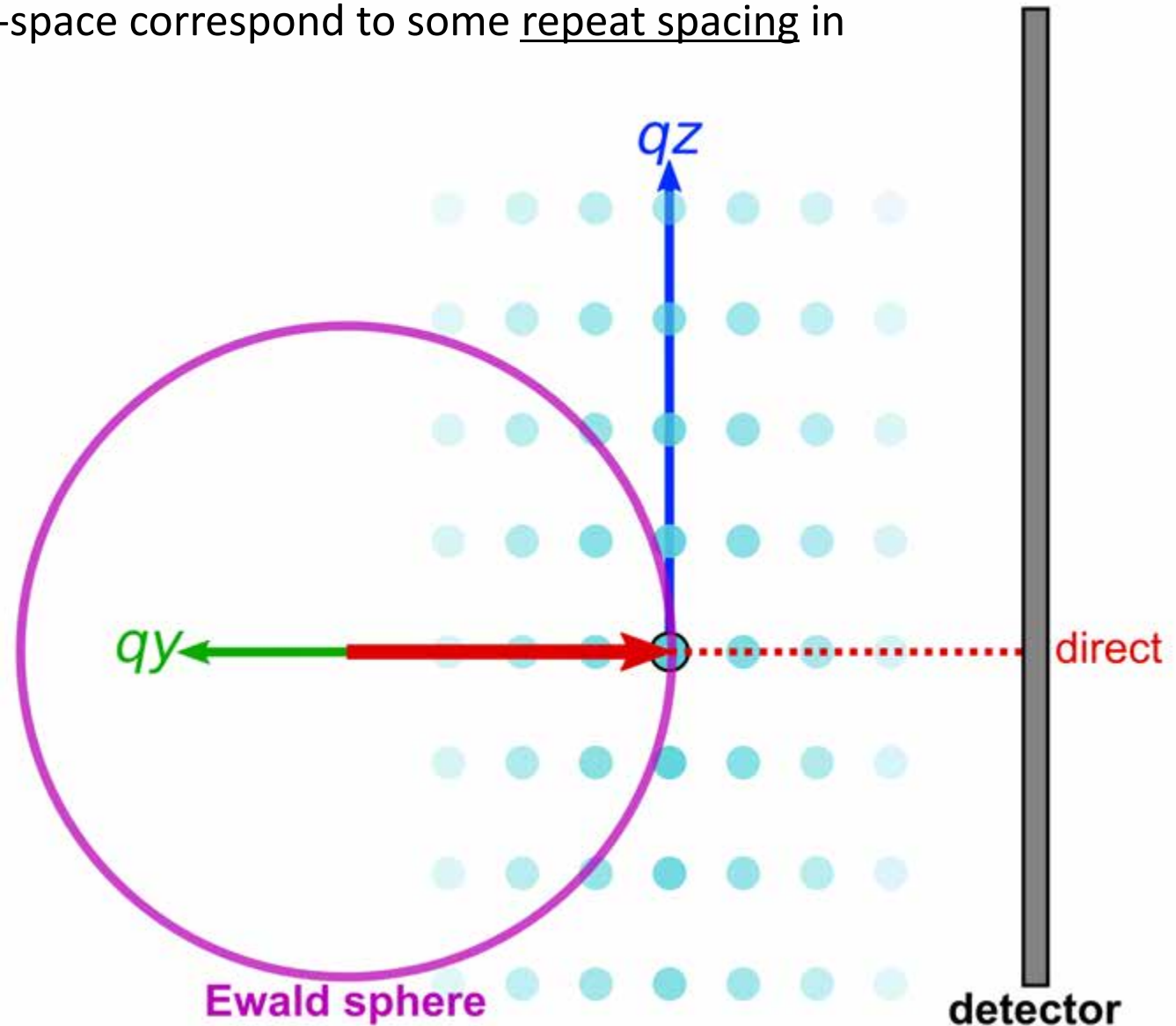
Detector records the intersection between the Ewald sphere and the reciprocal-space



For SAXS, it's almost a plane...

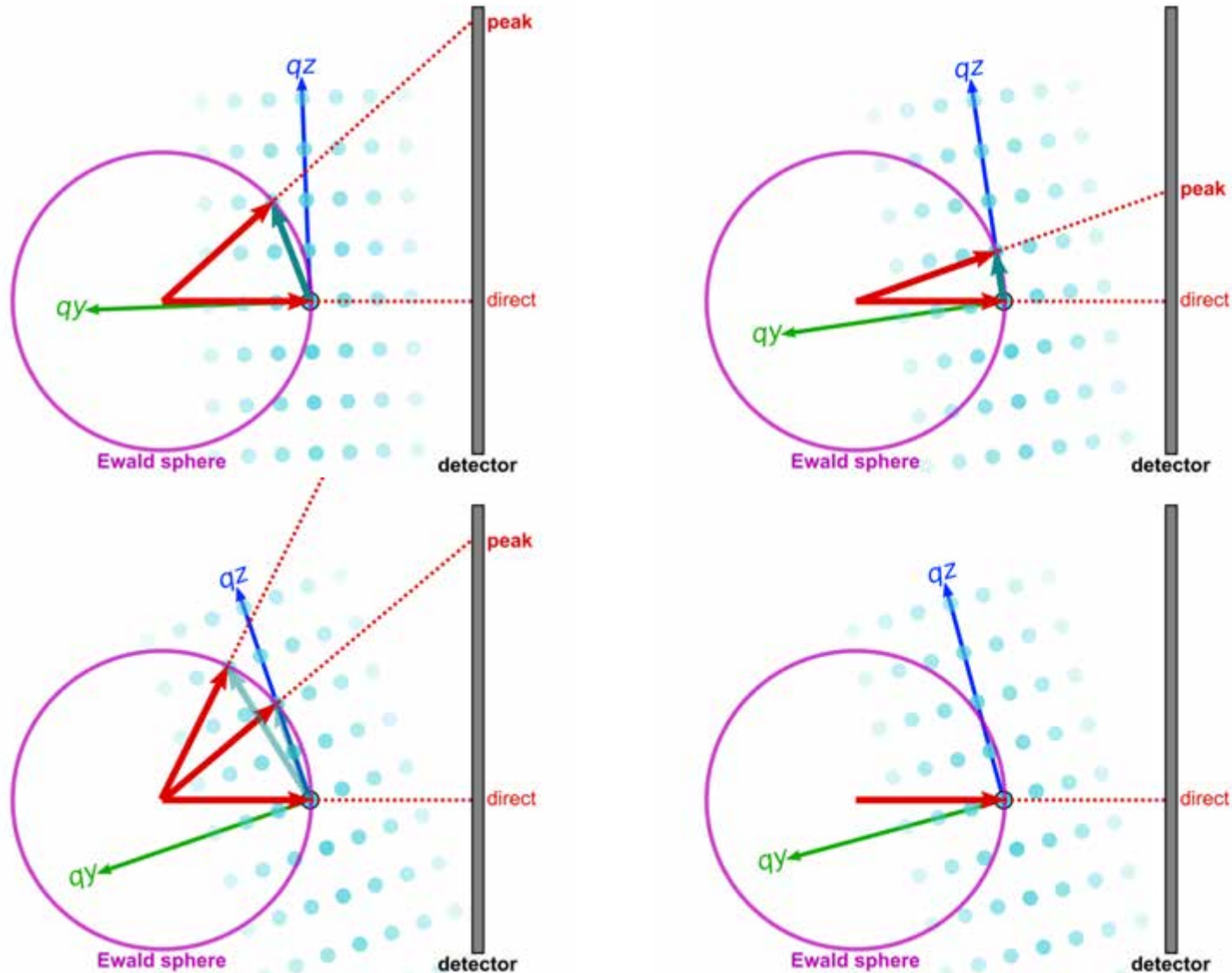
Reciprocal lattice

- Reciprocal-space is the Fourier transform of the realspace structure
- Peaks in reciprocal-space correspond to some repeat spacing in realspace
- $q = 2\pi/d$



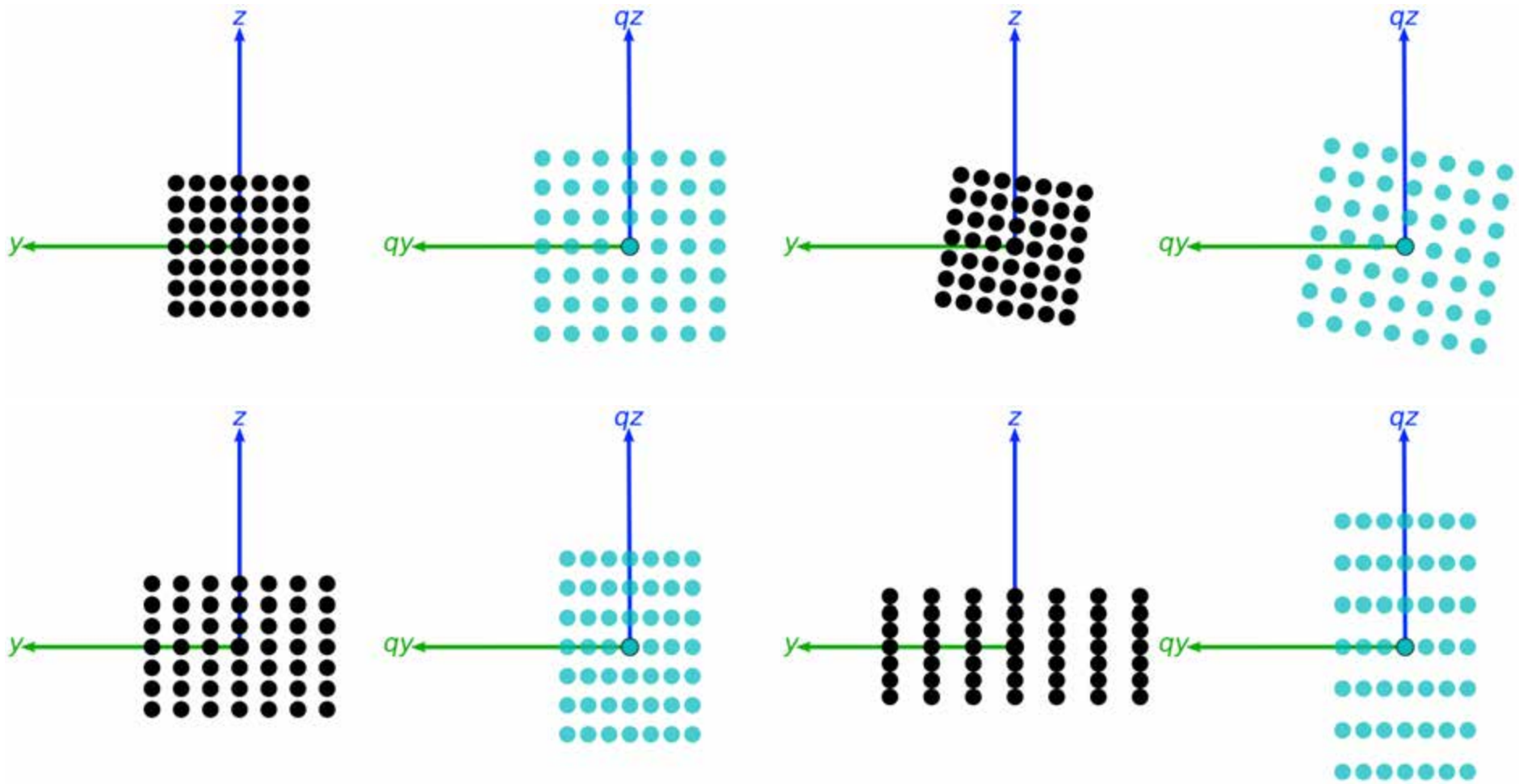
Reciprocal lattice

- You observe a peak on the detector when the reciprocal lattice is aligned such that the Ewald sphere intersects the peak



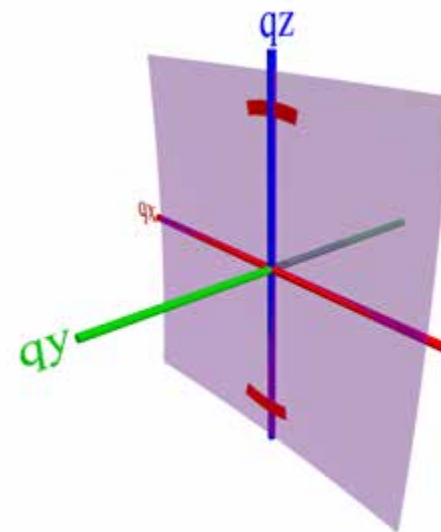
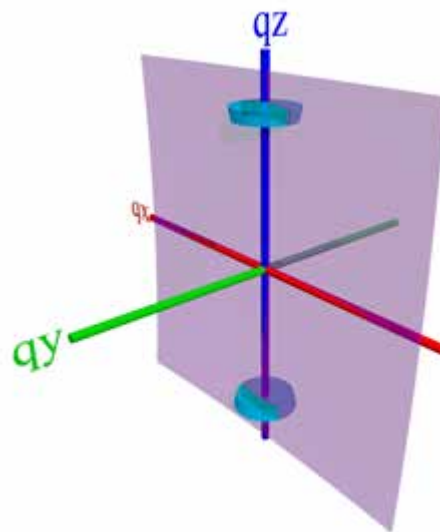
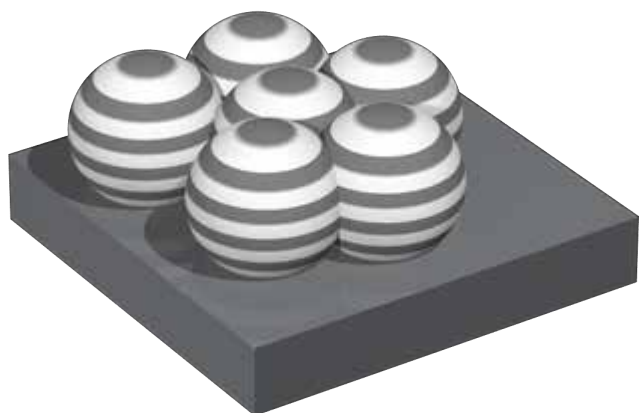
Reciprocal lattice examples

- Peaks come from repeating structures
- Distance is inverted: $d = 2\pi/q$

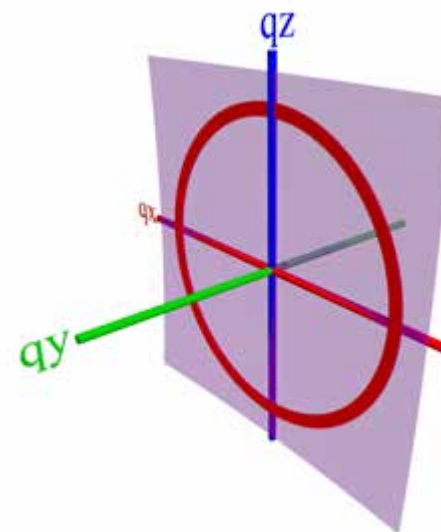
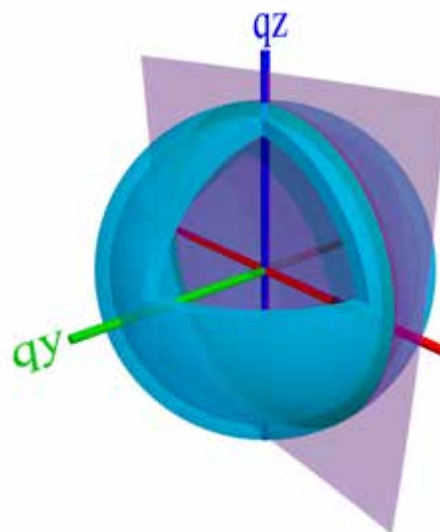
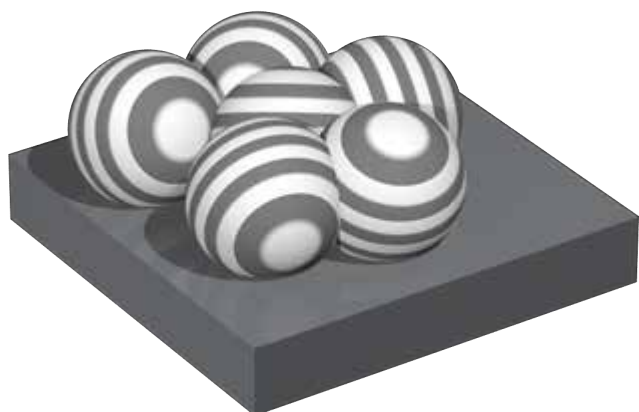


3D Reciprocal-space

- Aligned

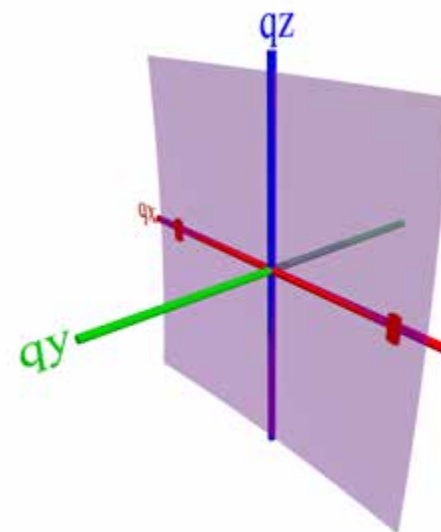
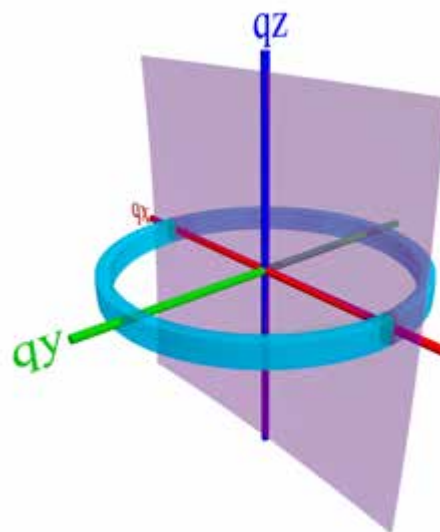
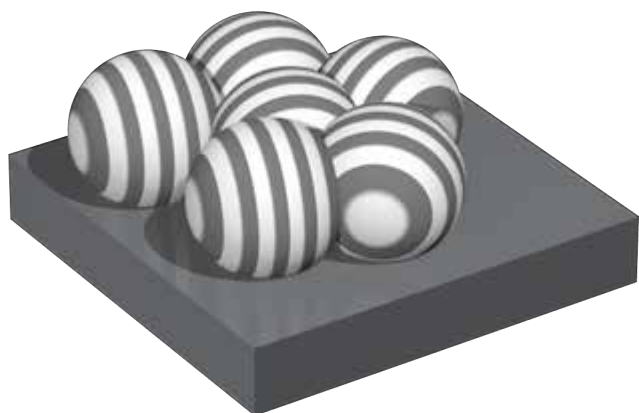


- Isotropic

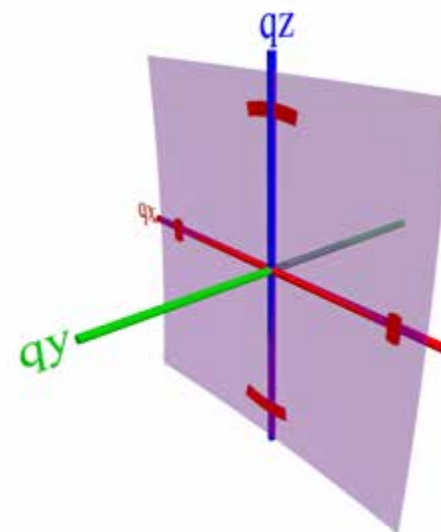
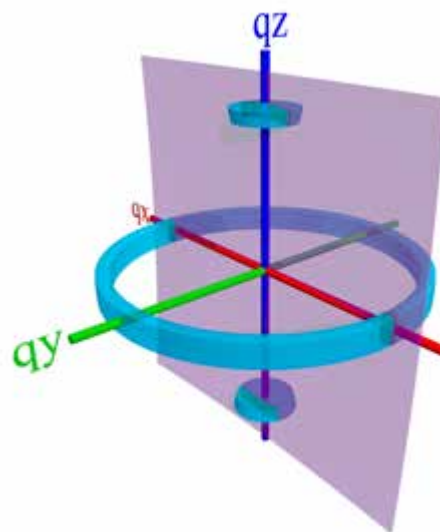
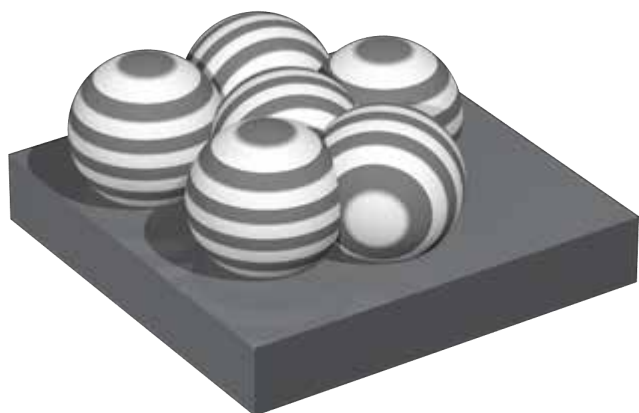


3D Reciprocal-space

- In-plane powder

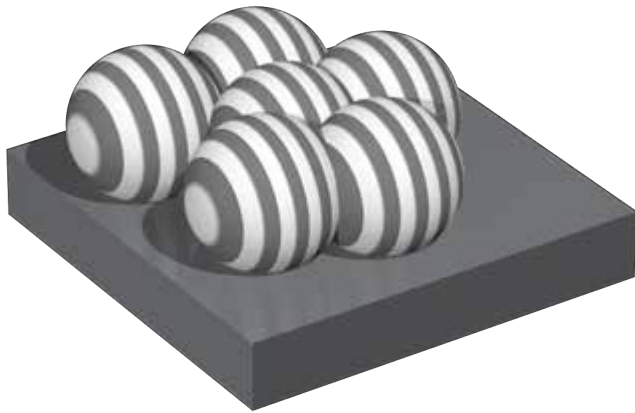


- Bimodal



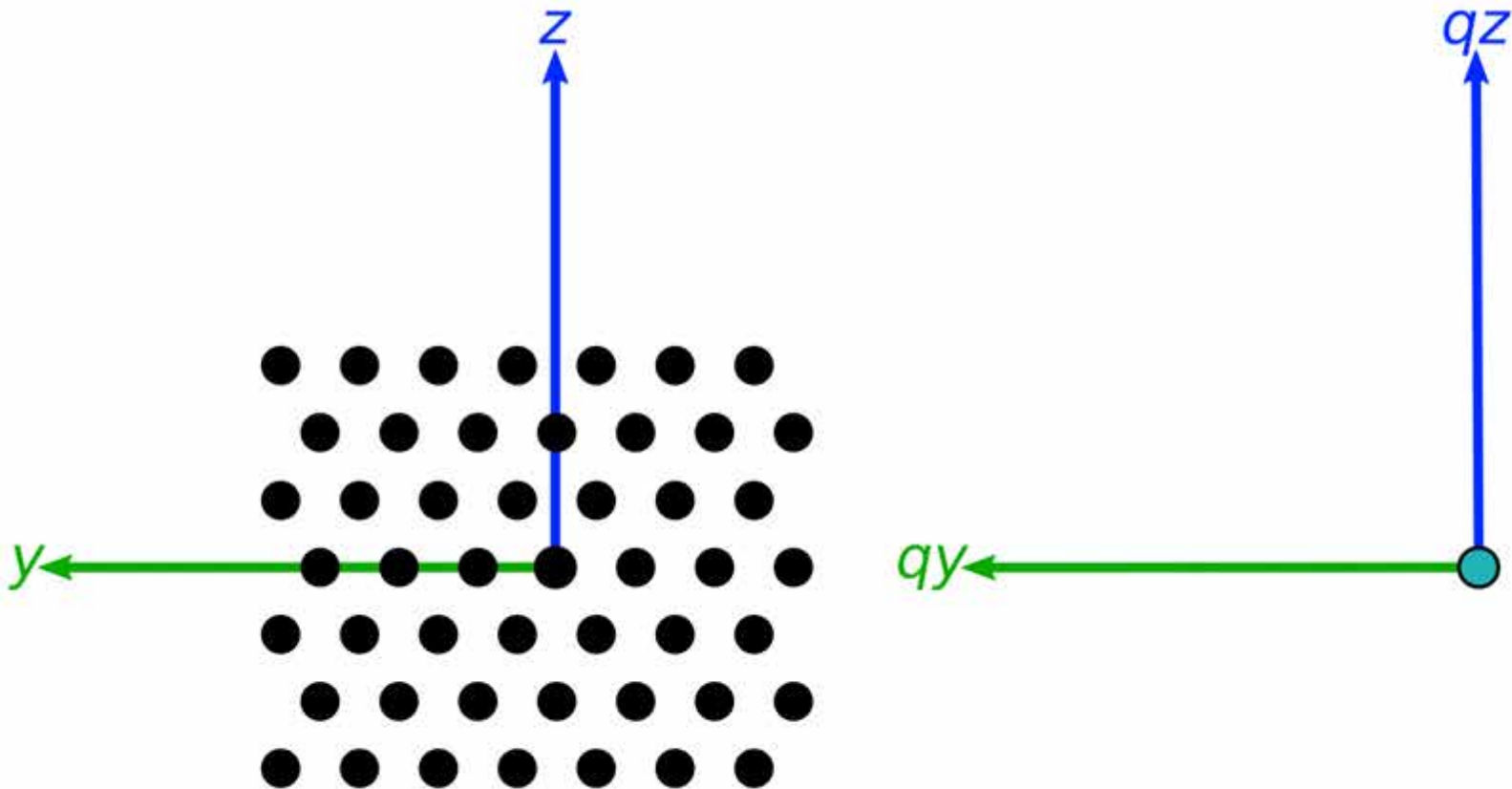
3D Reciprocal-space

- ?



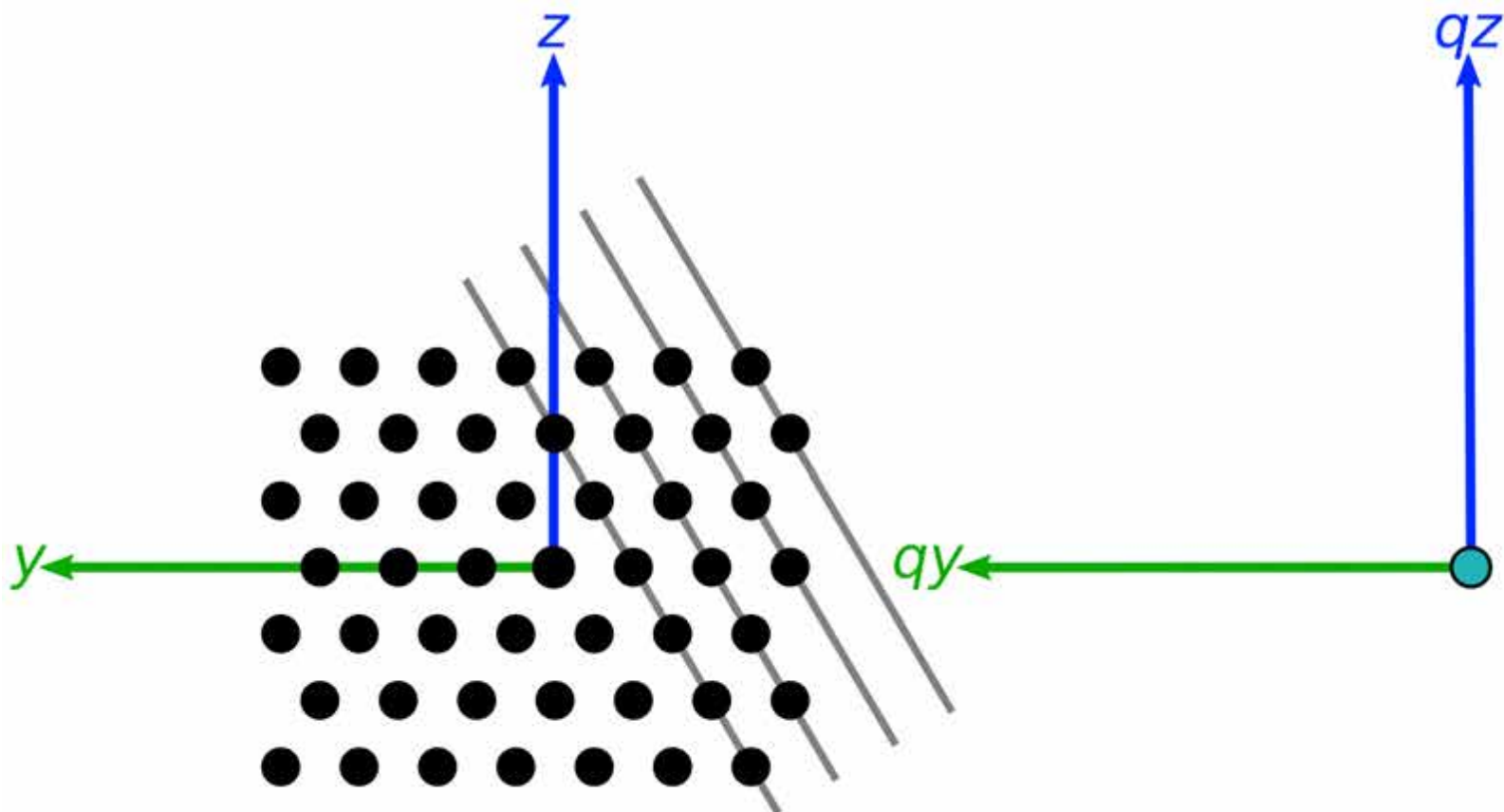
Reciprocal lattice exercise

- What lattice will be generated?



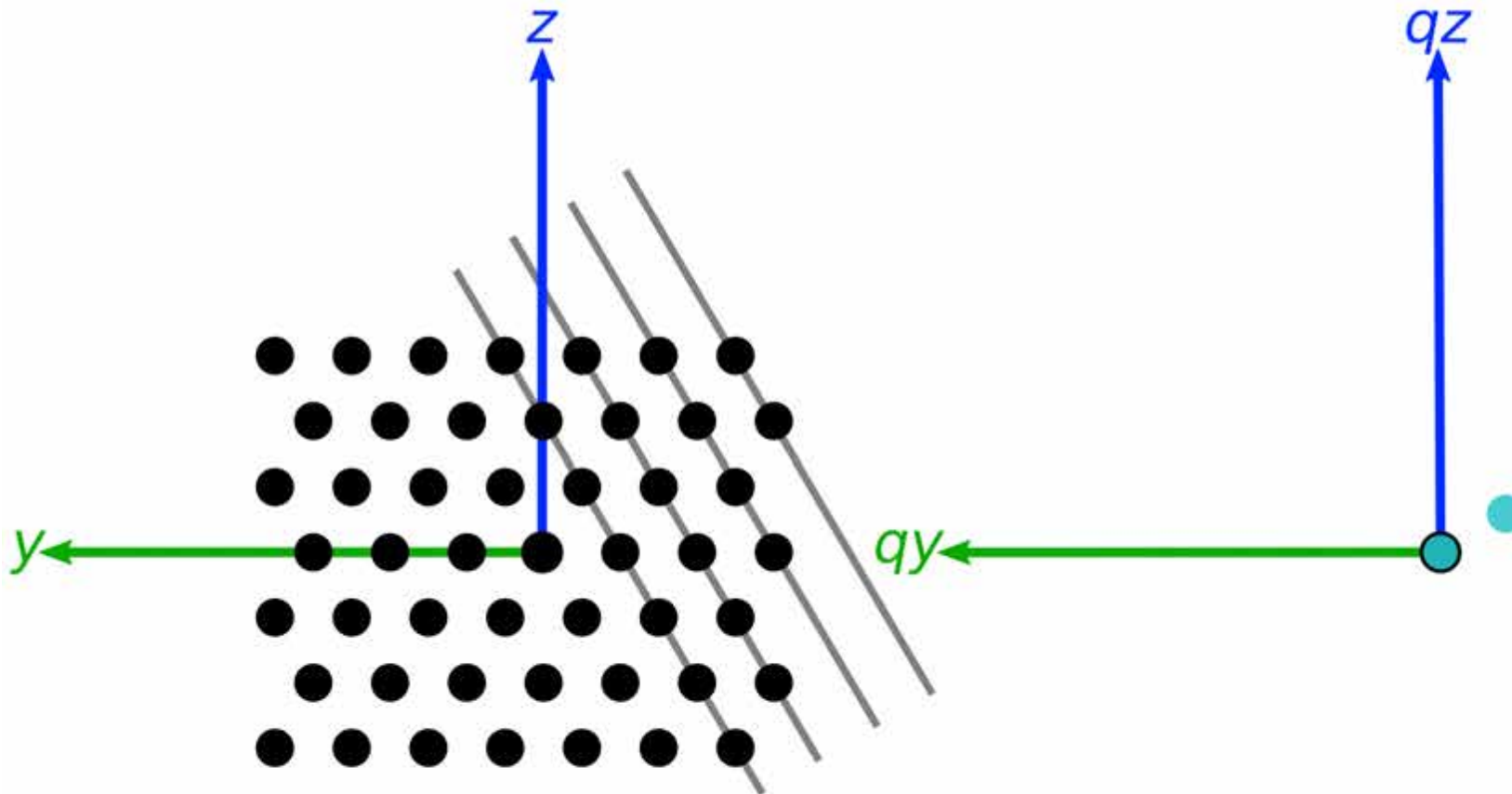
Reciprocal lattice exercise

- Identify realspace planes



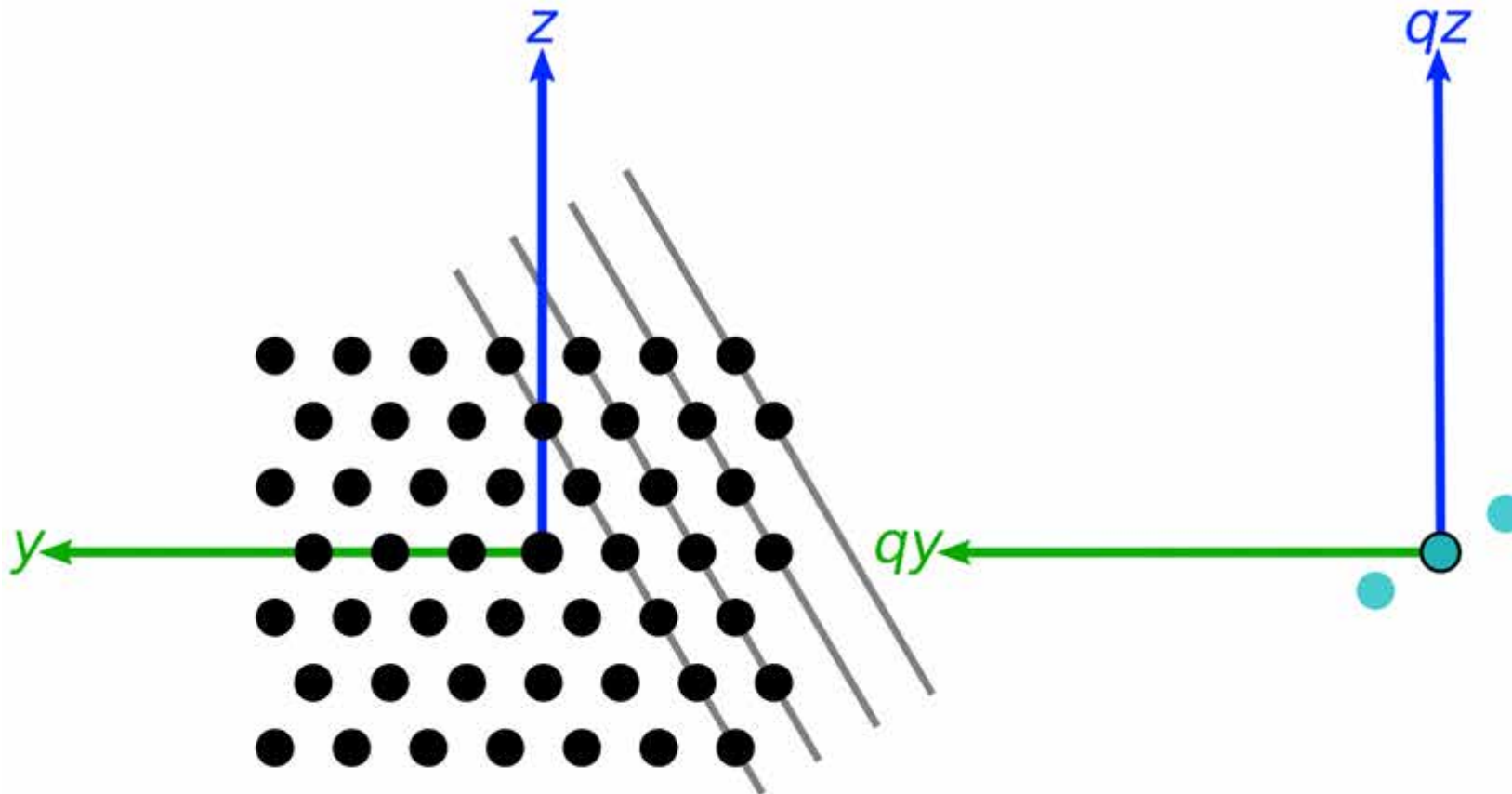
Reciprocal lattice exercise

- This repeating structure yields a peak normal to the planes



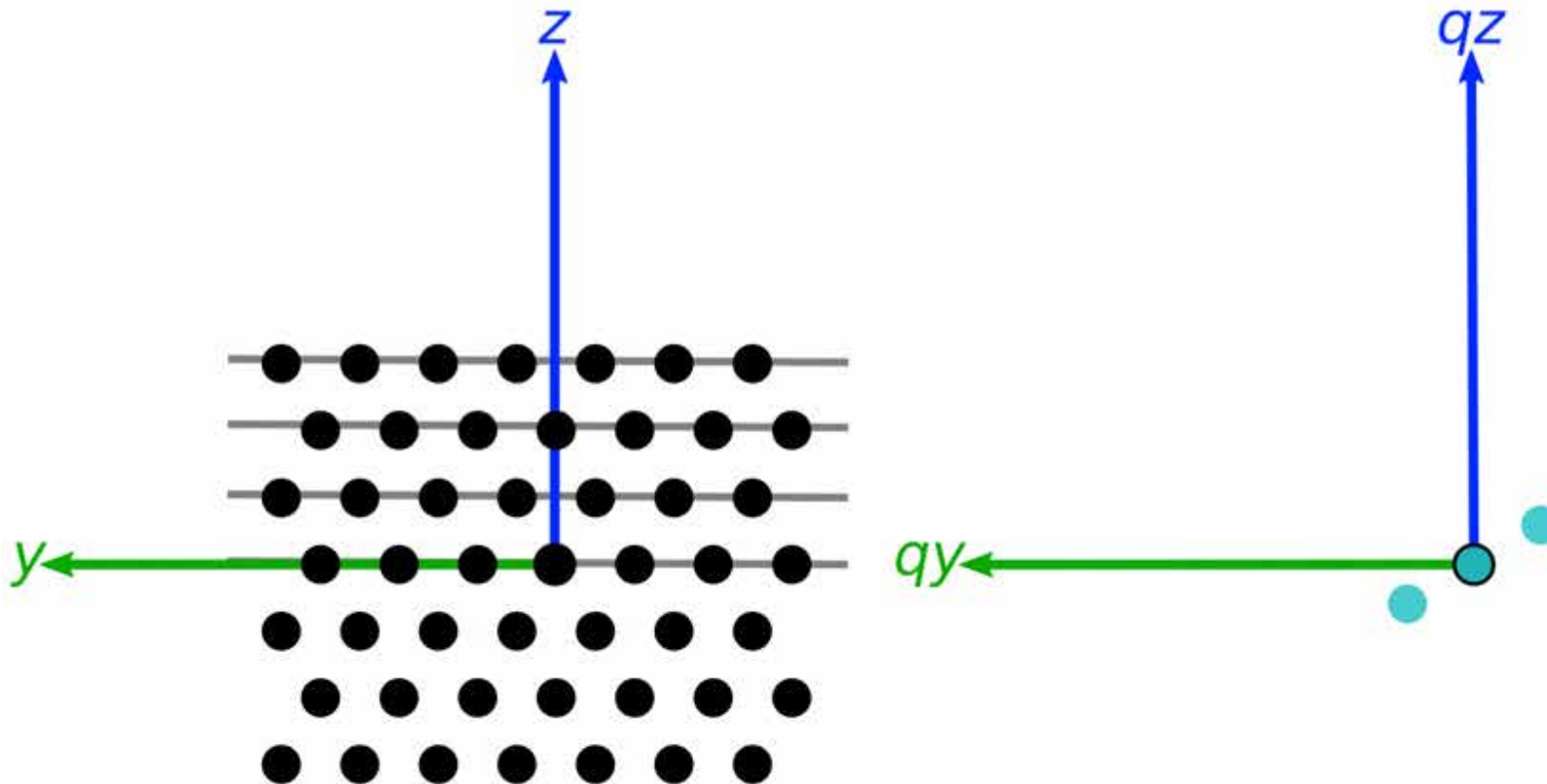
Reciprocal lattice exercise

- Reciprocal-space is centro-symmetric



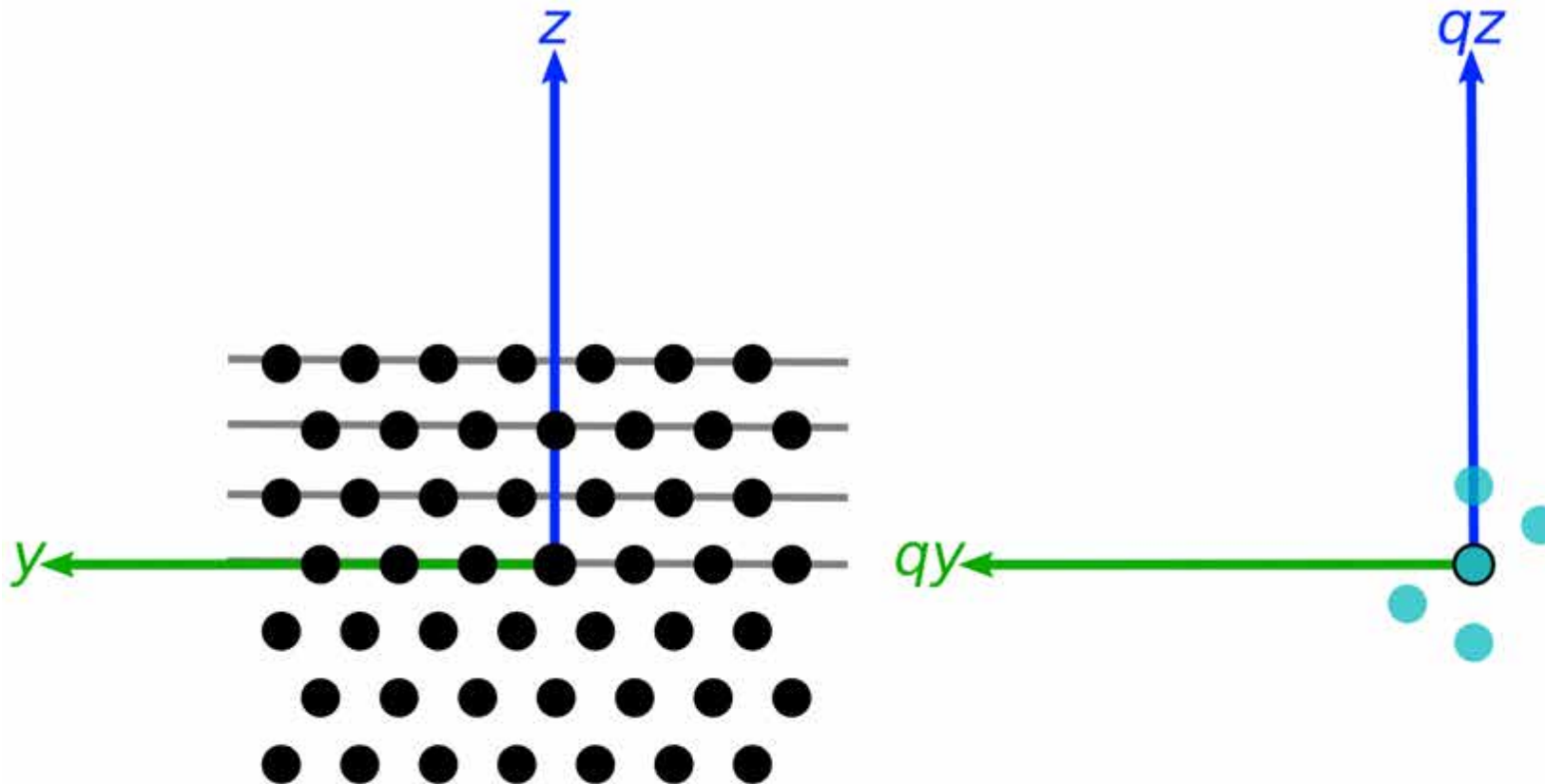
Reciprocal lattice exercise

- Another set of planes...



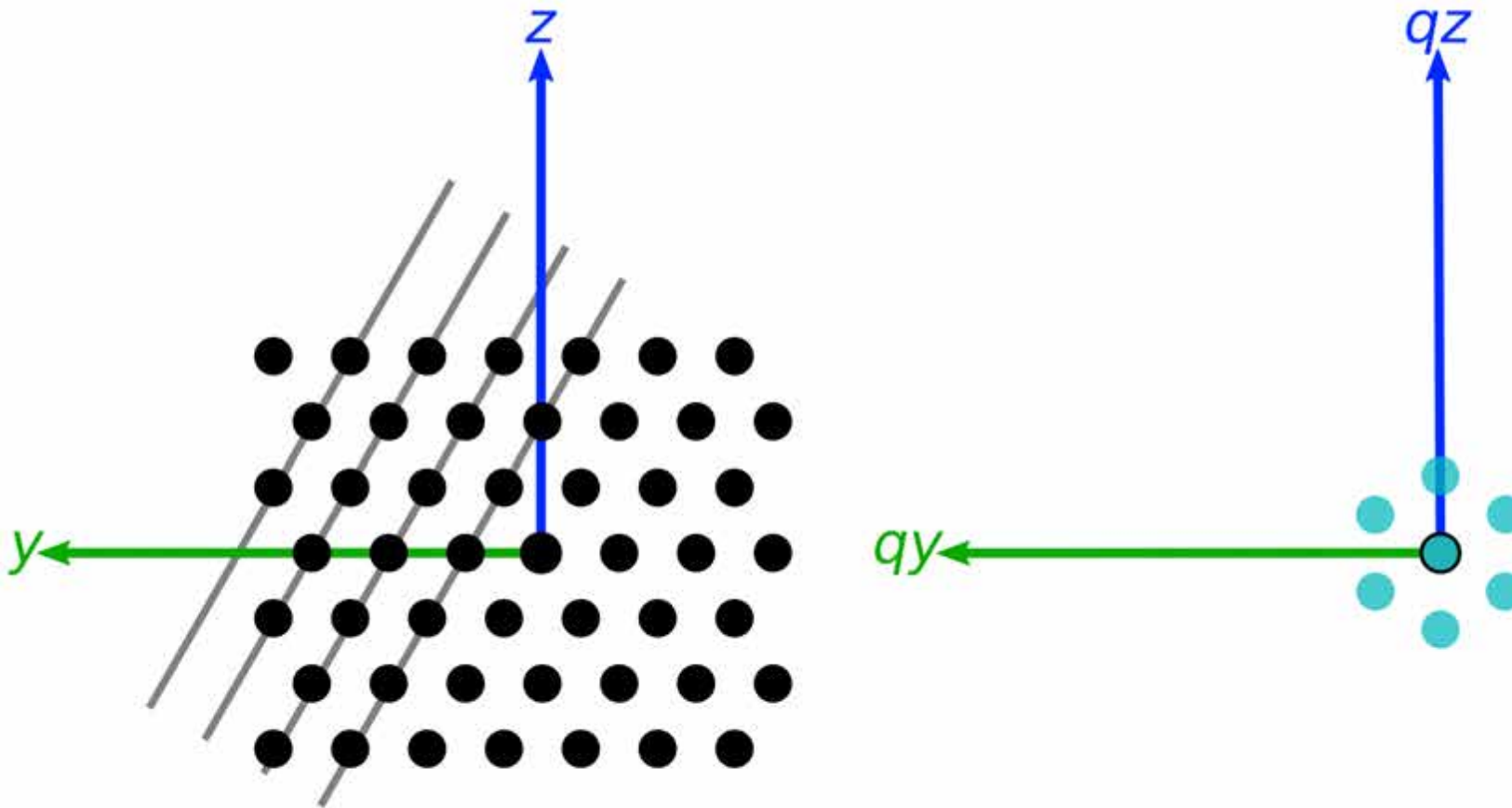
Reciprocal lattice exercise

- Another set of planes...
- ... another set of peaks.



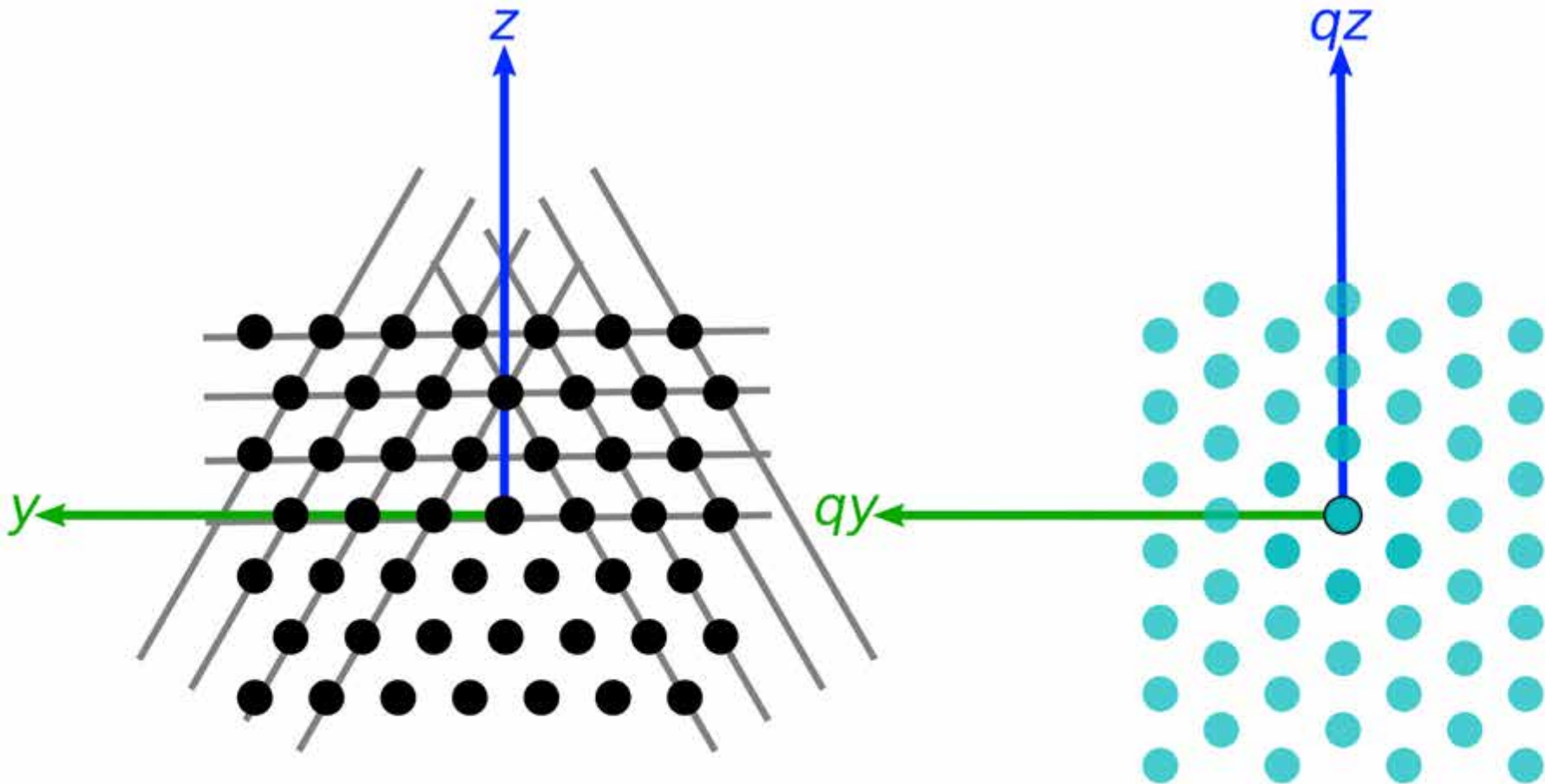
Reciprocal lattice exercise

- Reciprocal peaks are arranged in hexagonal lattice
- But note the orientation!
- Also: no peak corresponds to the particle-particle distance...



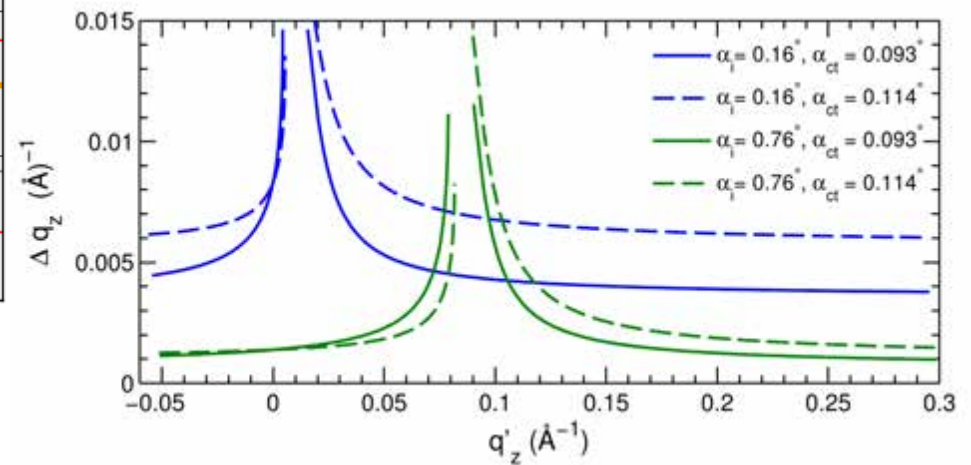
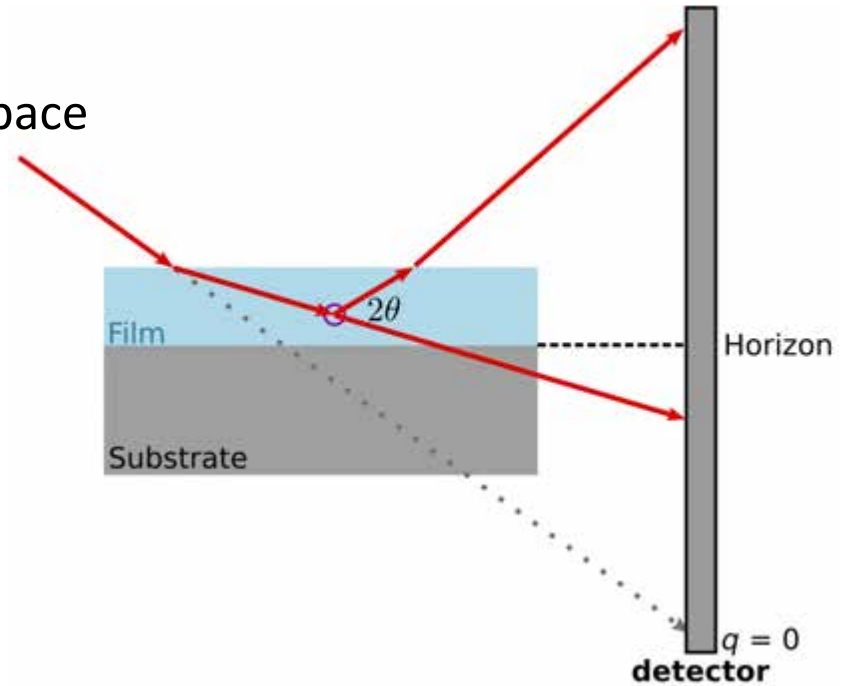
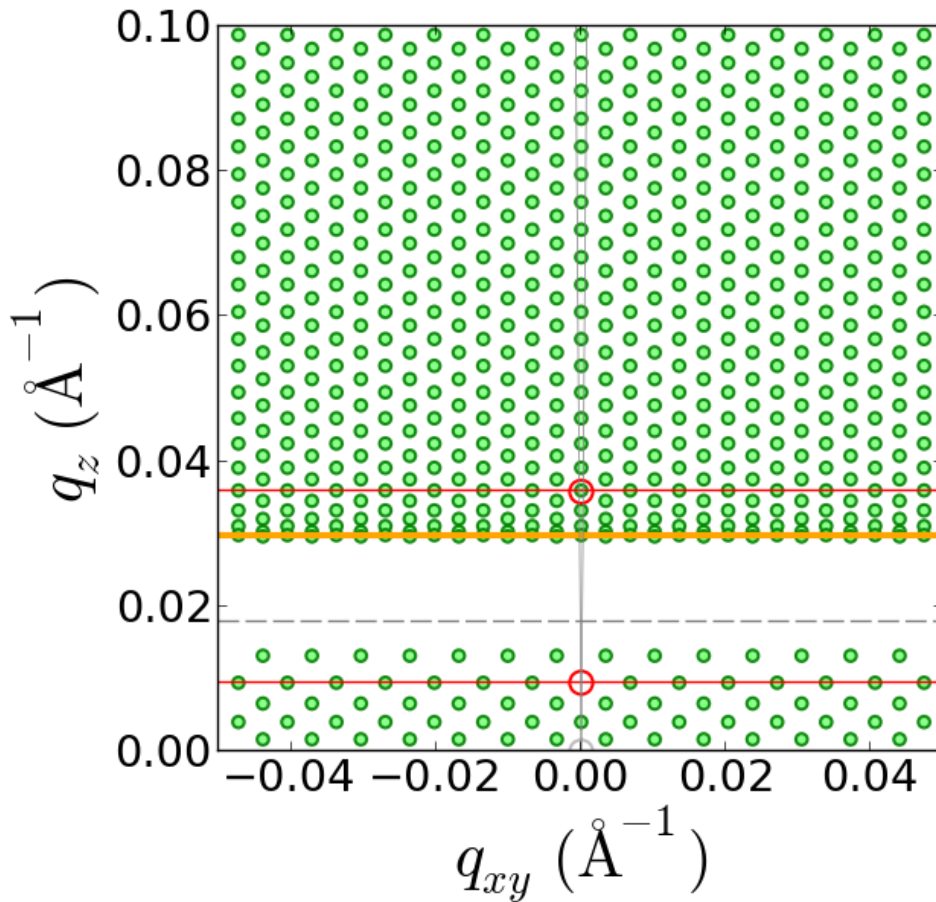
Reciprocal lattice exercise

- We also get higher-order peaks
- We've neglected peak intensities and shape, which encode all the details of the structure (finite size \rightarrow peak width, electron density distribution \rightarrow peak intensities, disorder \rightarrow peak falloff, ...)



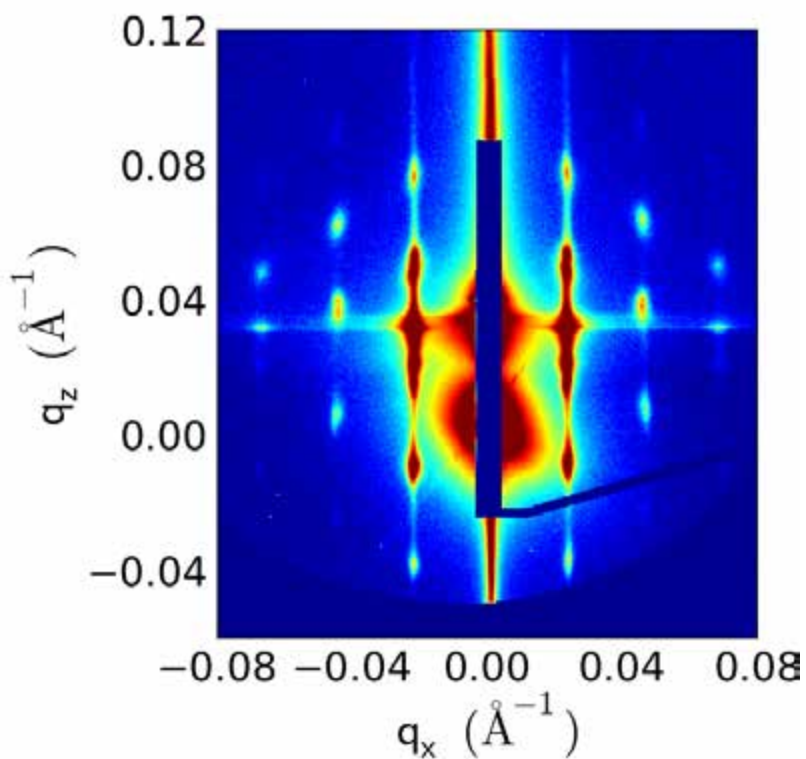
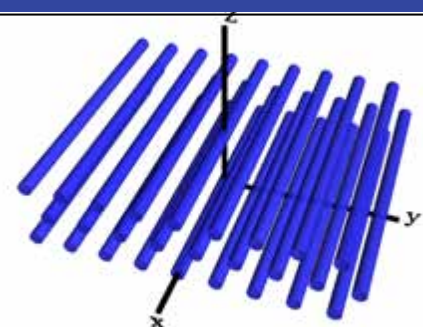
GISAXS

- GISAXS makes this more complicated:
 - Refraction shifts and distorts reciprocal-space

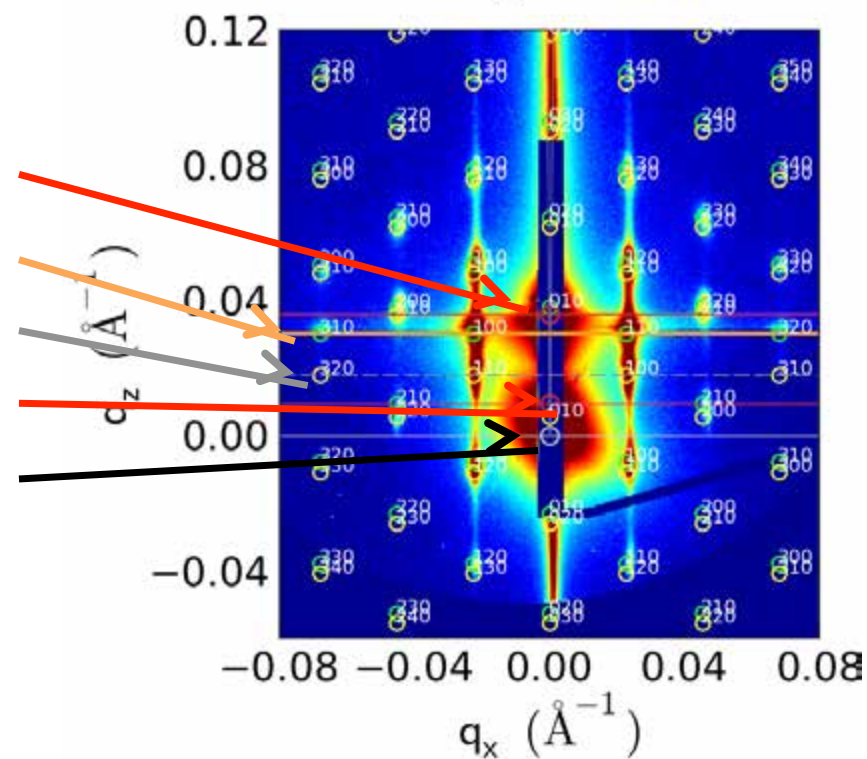


GISAXS

- GISAXS makes this more complicated:
 - Refraction shifts and distorts reciprocal-space
 - Reflection leads to two sets of peaks



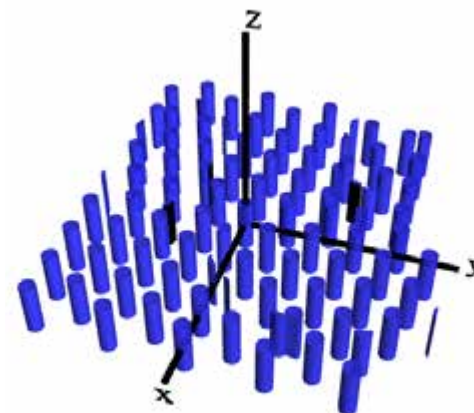
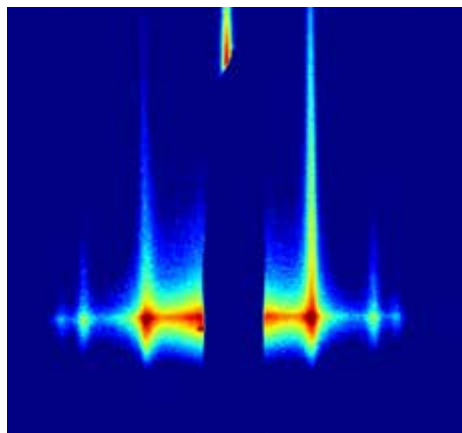
specular
Yoneda
horizon
refracted
direct beam



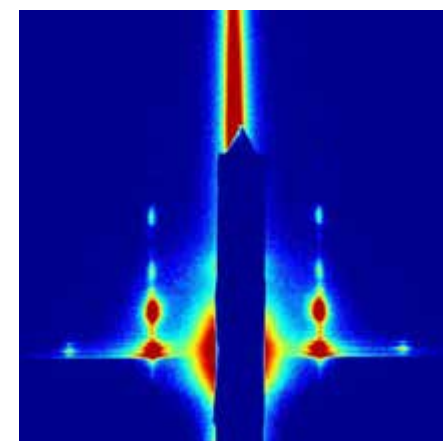
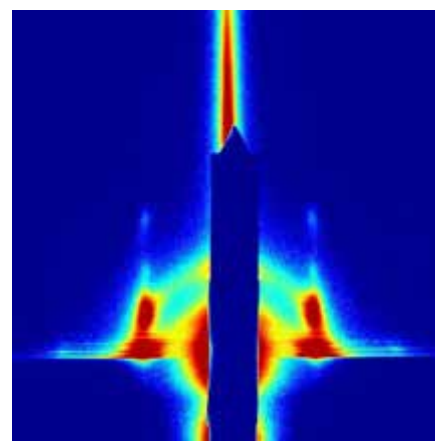
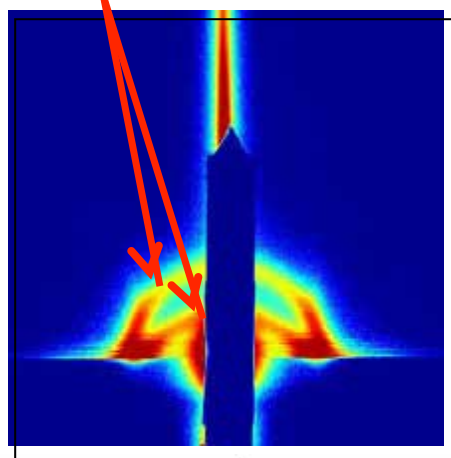
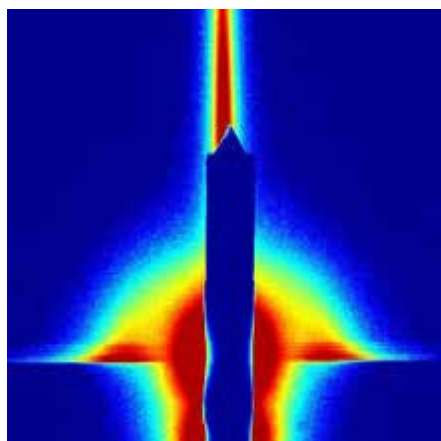
- You can use DWBA to predict peak positions
- So, you can index all the peaks, ...

Examples

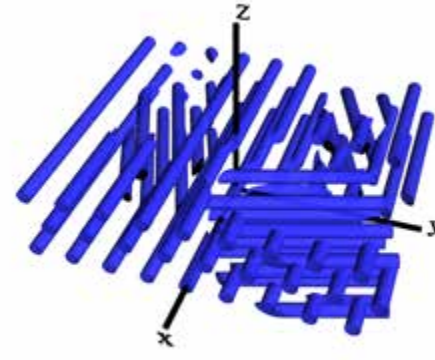
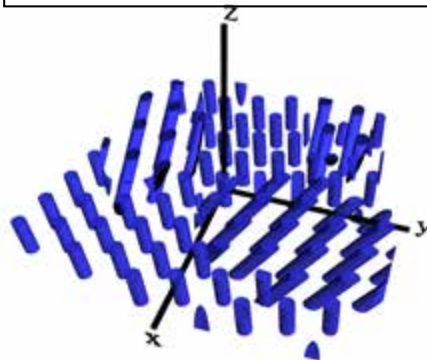
- Block-copolymer cylinder phase



Note two rings

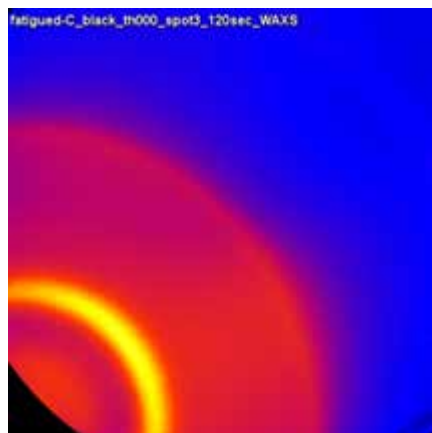


Disordered

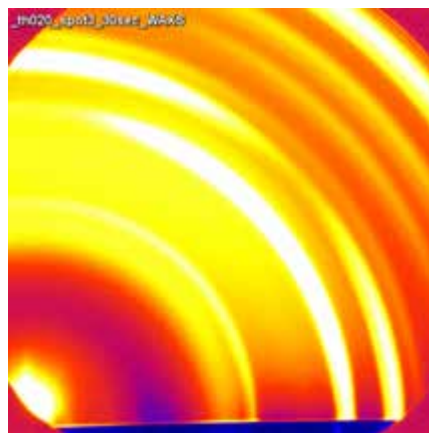


Qualitative

- Diffuse scattering comes from disorder, roughness...
- Halo usually means amorphous, sharp rings means good order (crystal?)
- Intensity along ring tells orientation
- Peaks becomes speckled as the grain size becomes very large
- An array of distinct peaks means crystal is well-oriented w.r.t. substrate
- If the peaks appear/disappear when you rotate, you may have a single crystal



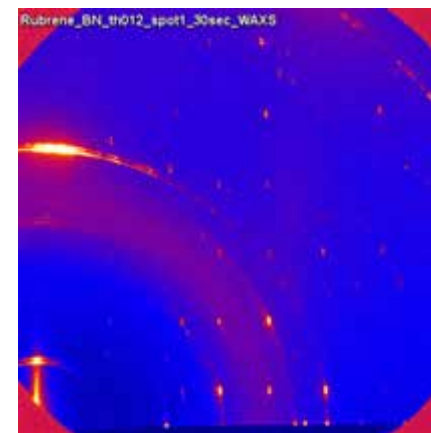
disordered



some ordering



oriented, textured

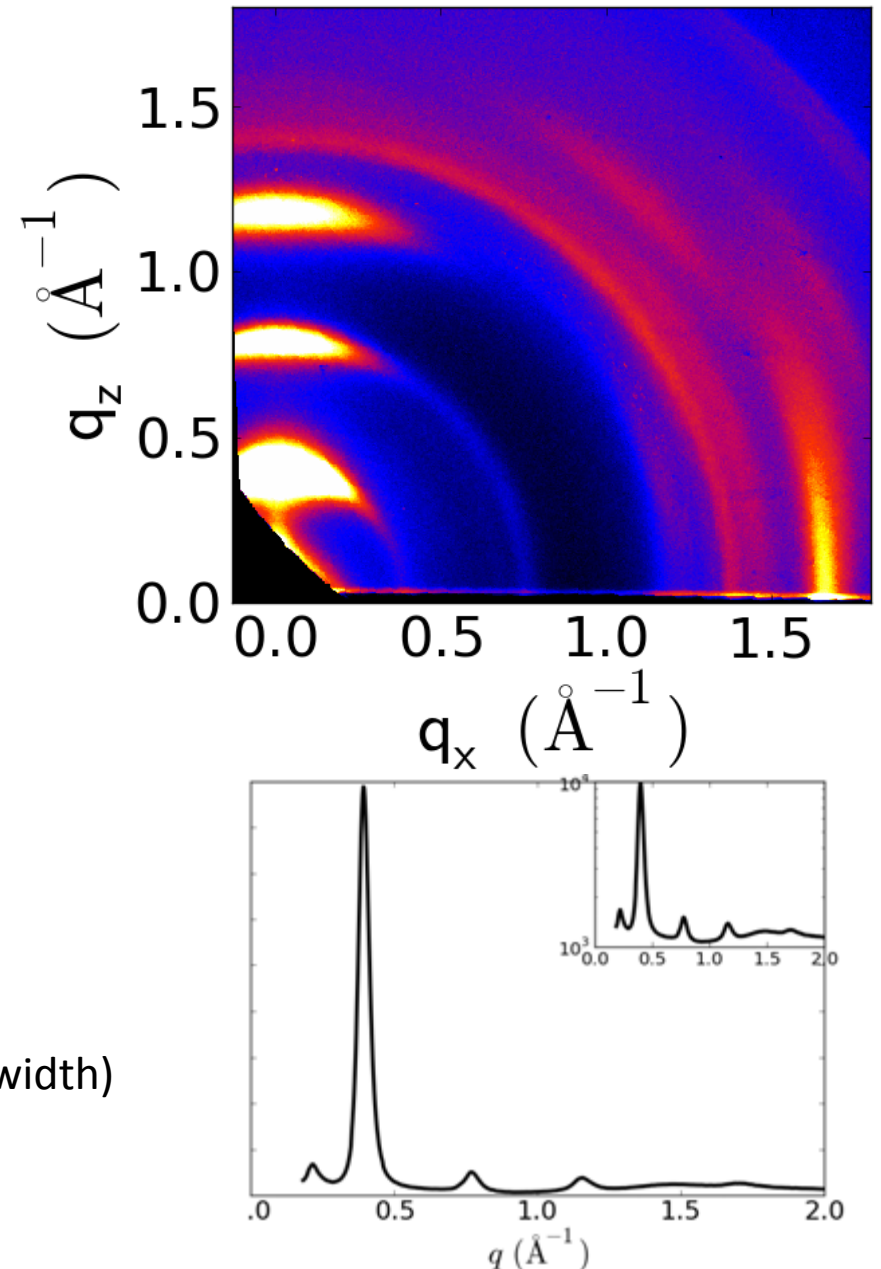


single crystal



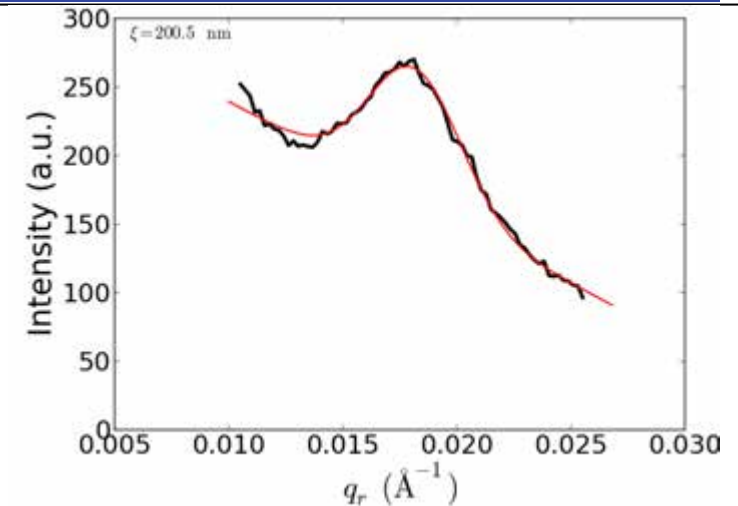
Linecut

- Calibration converts from pixel to q
 - Detector distance
 - X-ray energy
 - Beam position
 - Detector orientation/tilt
- Adjust intensity
 - Beam flux and measurement time
 - Polarization, pixel acceptance, ...
- Linecuts:
 - 1D radial average
 - Radial (in-plane, out-of-plane, other)
 - “Straight” (q_z or q_r)
 - Along an arc
- Binning:
 - Average multiple pixels to improve SNR
 - Don't smear-out real features! (e.g. peak width)
 - Account for background!



Linecut: what can it tell you?

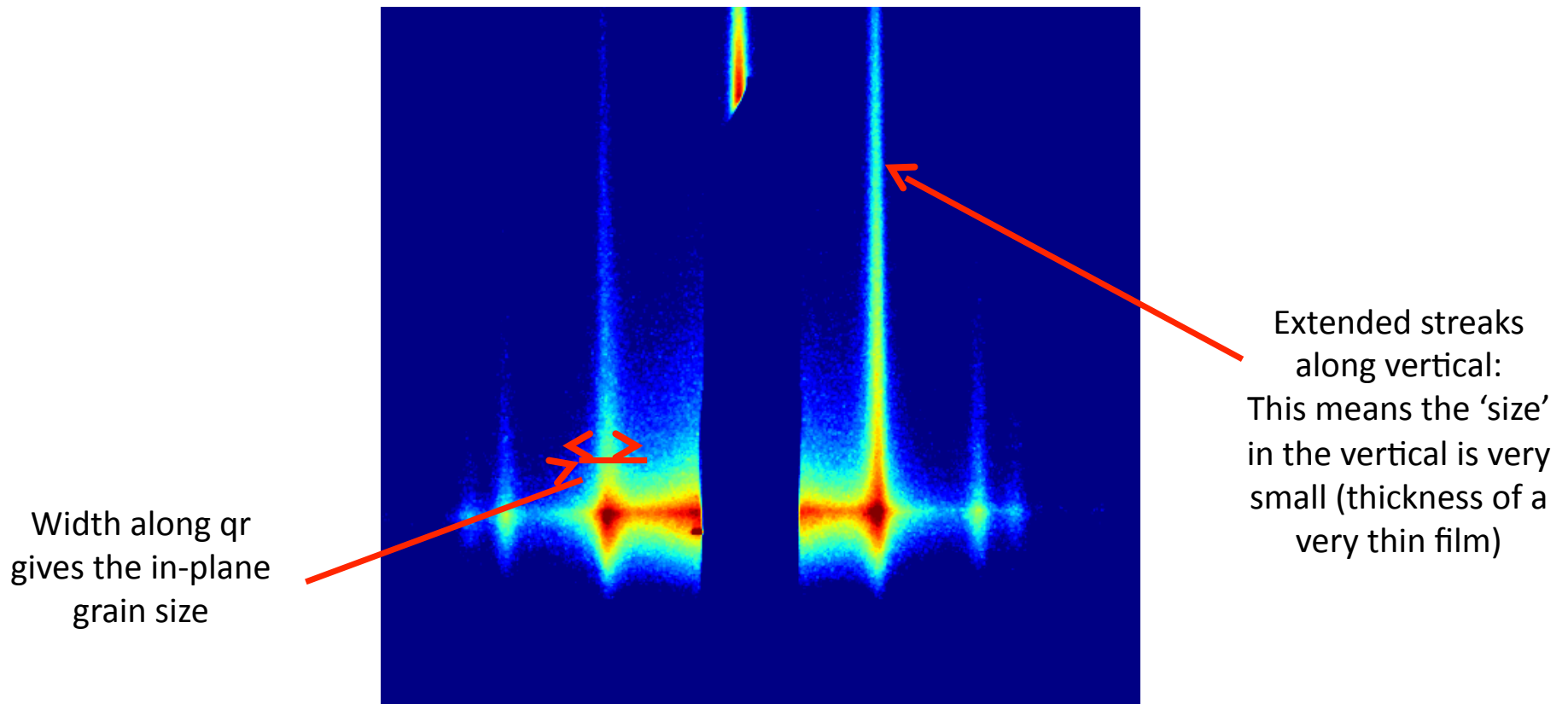
- q -position $\rightarrow d = 2\pi/q$
 - But don't forget about unit-cell symmetry!
- q -positions \rightarrow Unit cell symmetry, dimensions and distortion
- Peak width $\rightarrow \xi = 2\pi K/\Delta q$
 - Must account for instrumental resolution
 - Many things can affect peak width (microstrain, faults, ...)
- Intensity \rightarrow Population analysis (amount of material)
 - Must account for various factors: peak multiplicity, sample symmetry, experimental (flux, acceptance), ...
 - Comparing between samples can be tricky (sample size/thickness, e^- contrast)
 - Use relative comparisons or internal standards



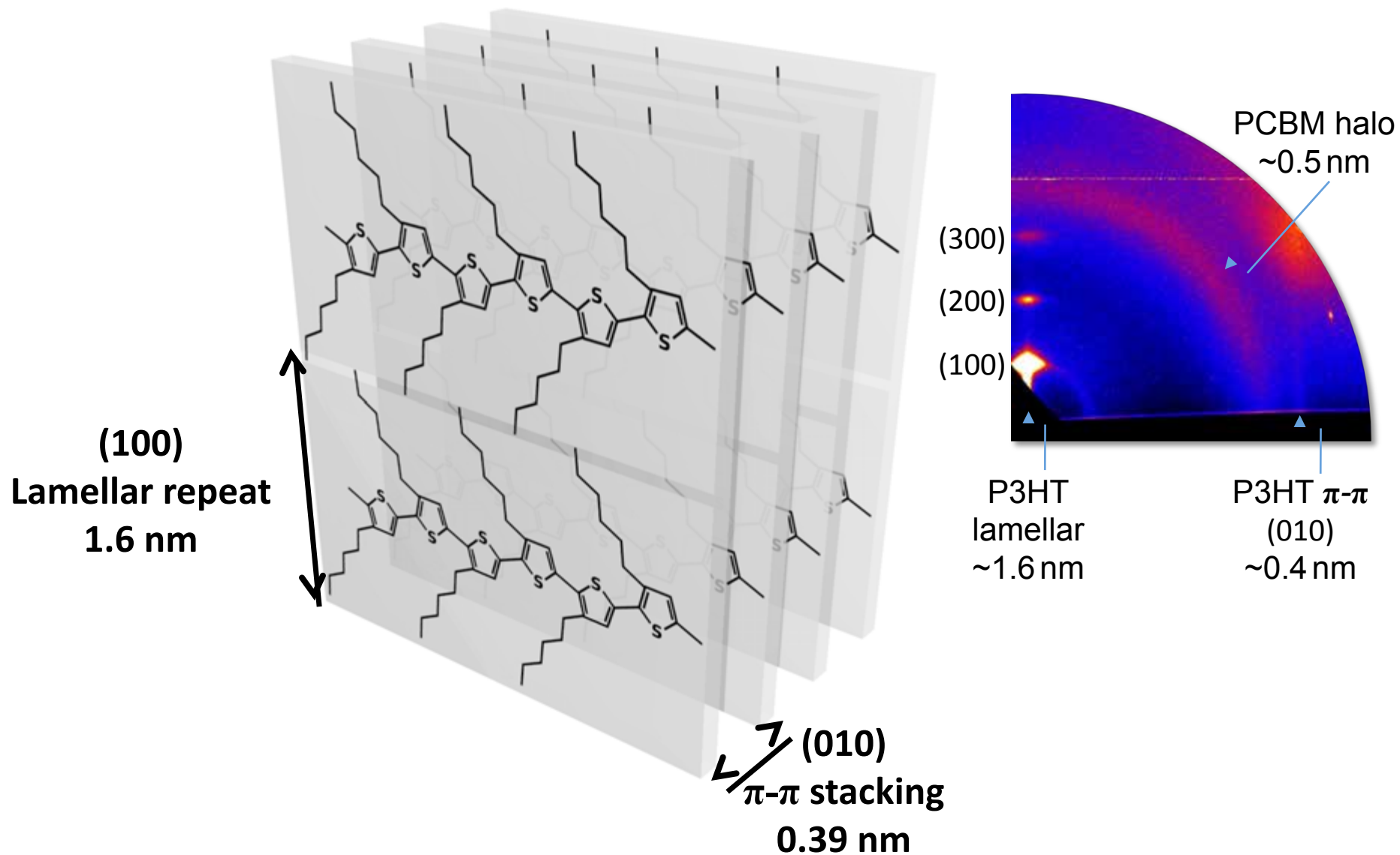
Smilgies *J. Appl. Cryst.*
2009, 42, 1030

Peak width

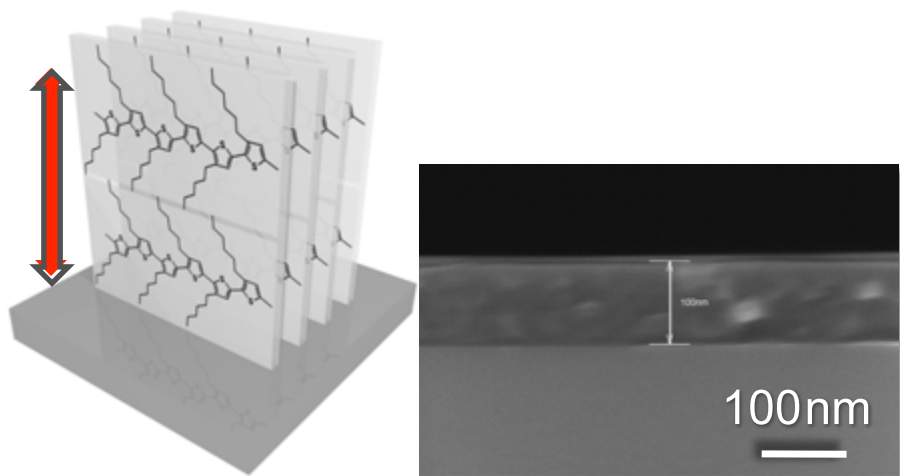
- Peak width can be anisotropic:
 - Width in each direction is telling you the 'grain size' in that direction
- Remember: distance is inverted $\xi = 2\pi K/\Delta q$
 - Narrow peak \rightarrow big grains
 - Broad peak \rightarrow small grains



P3HT: poly 3-hexylthiophene

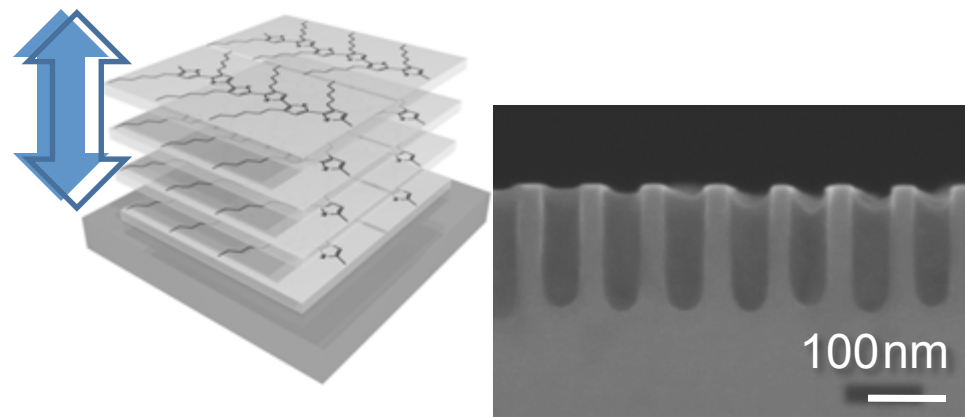


P3HT orientation



Edge-on

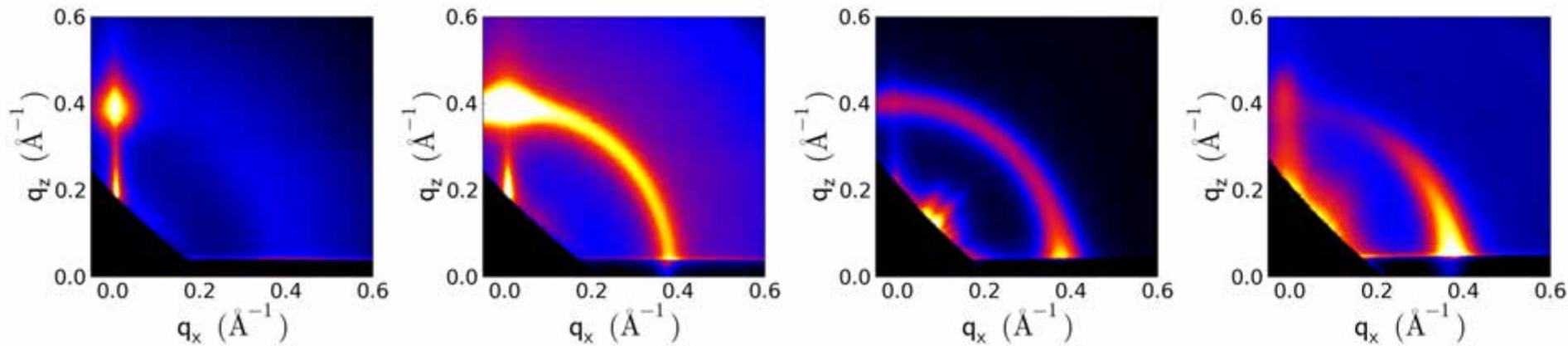
$$\alpha_h \approx 0.0002 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$$



Face-on

$$\alpha_h \approx 0.1 \text{ cm}^2\text{V}^{-1}\text{s}^{-1}$$

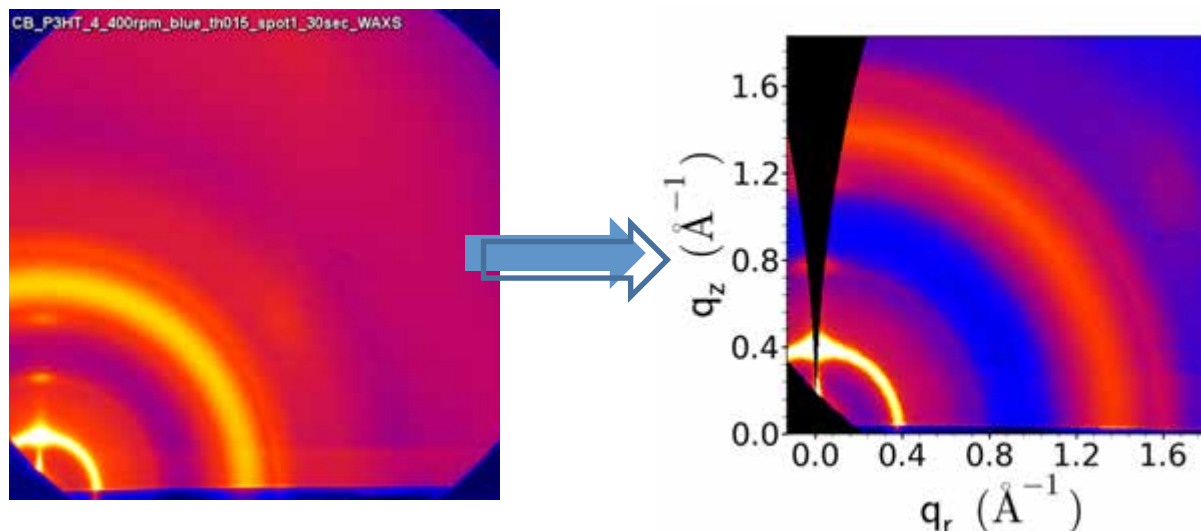
- Intensity along ring tells orientation



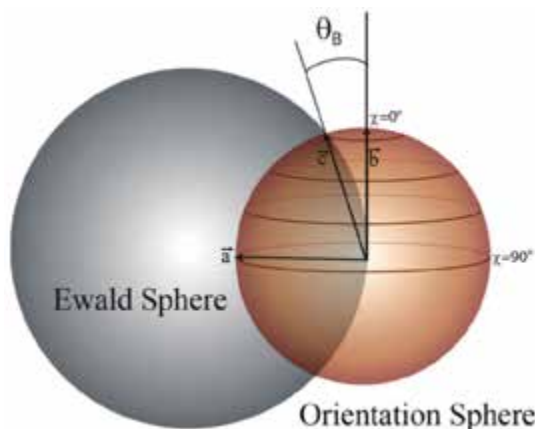
- How to quantify?

Analysis

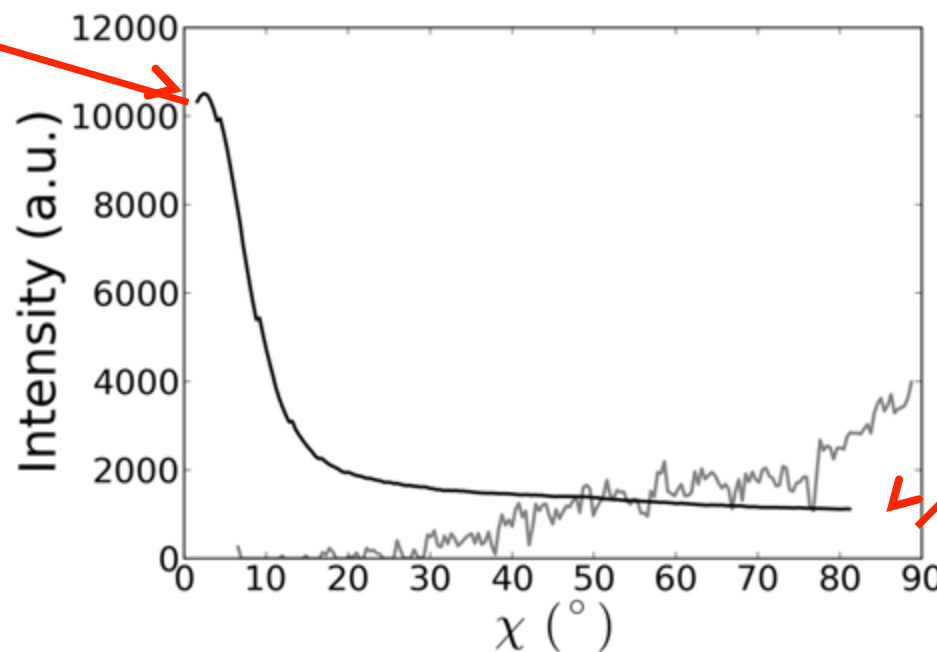
- Account for Bragg angle
- Think in terms of how the Ewald sphere intersects reciprocal-space
- Subtract background



Data doesn't start at 0



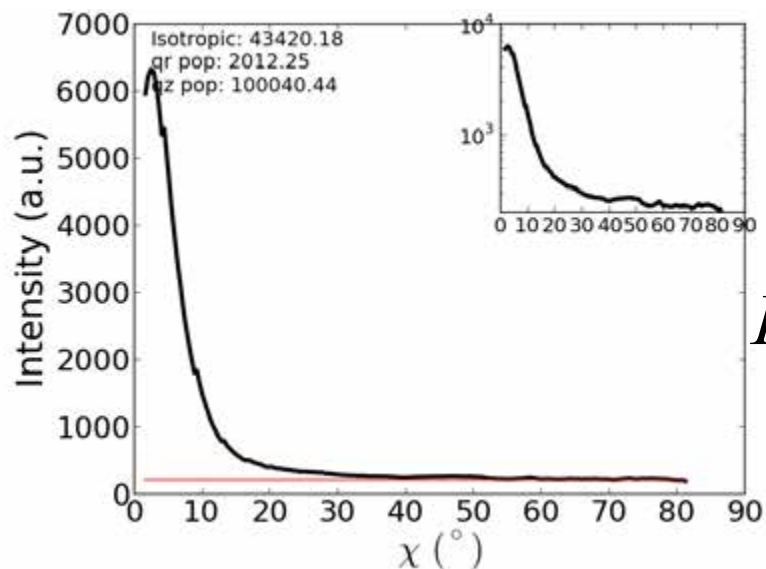
Baker *Langmuir* **2010**, 26, 9146



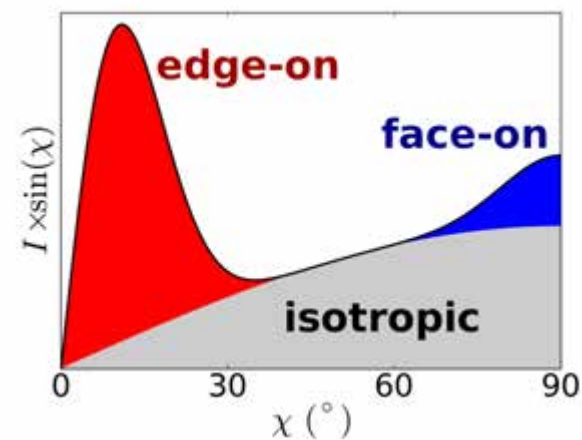
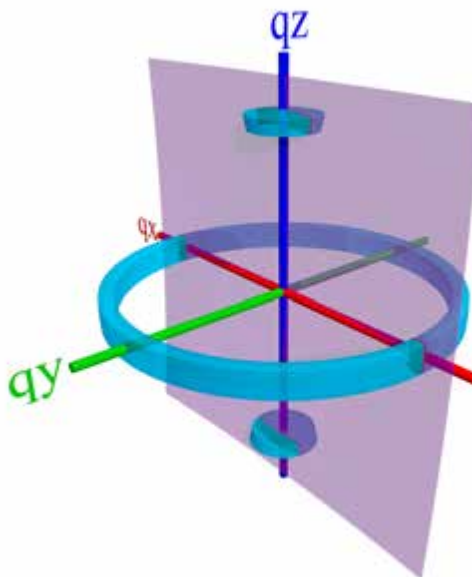
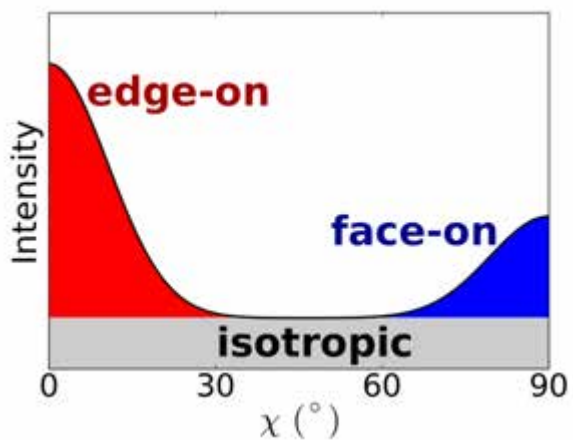
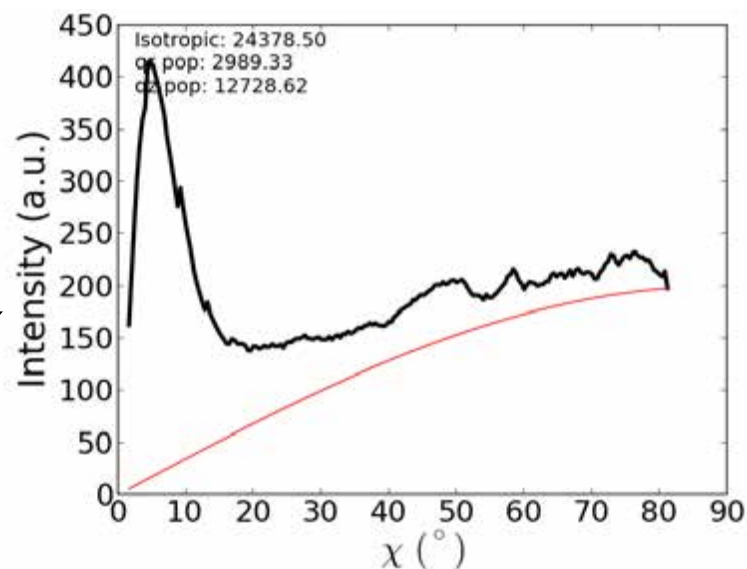
Exclude
Yoneda and
sub-horizon
data

Analysis

- Account for in-plane powder symmetry

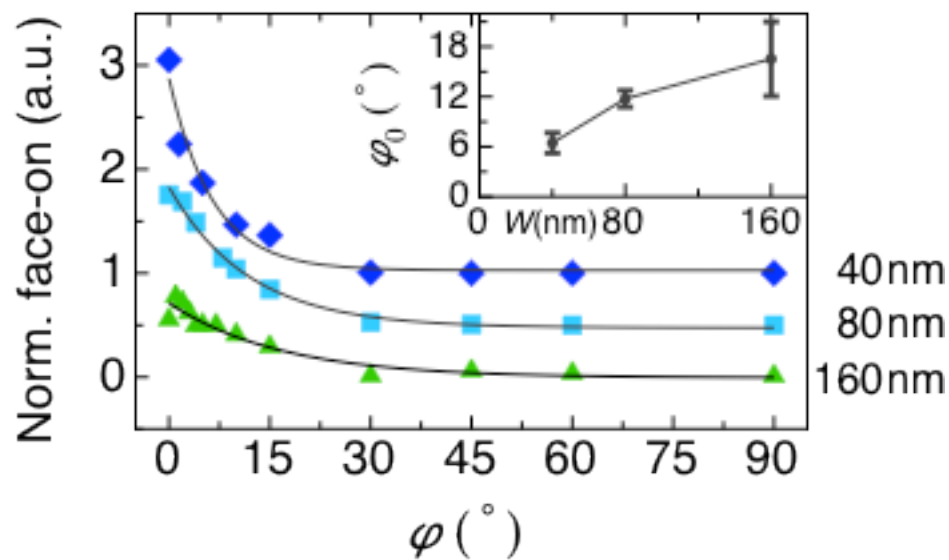
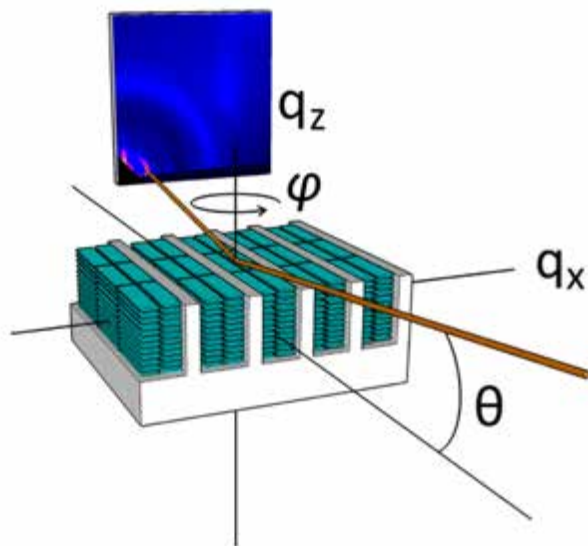
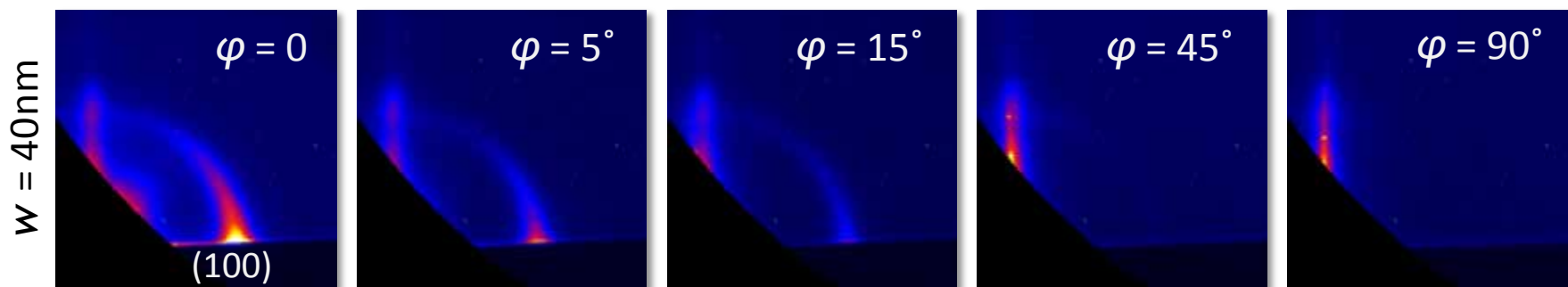


$I_c = I \times \sin \chi$



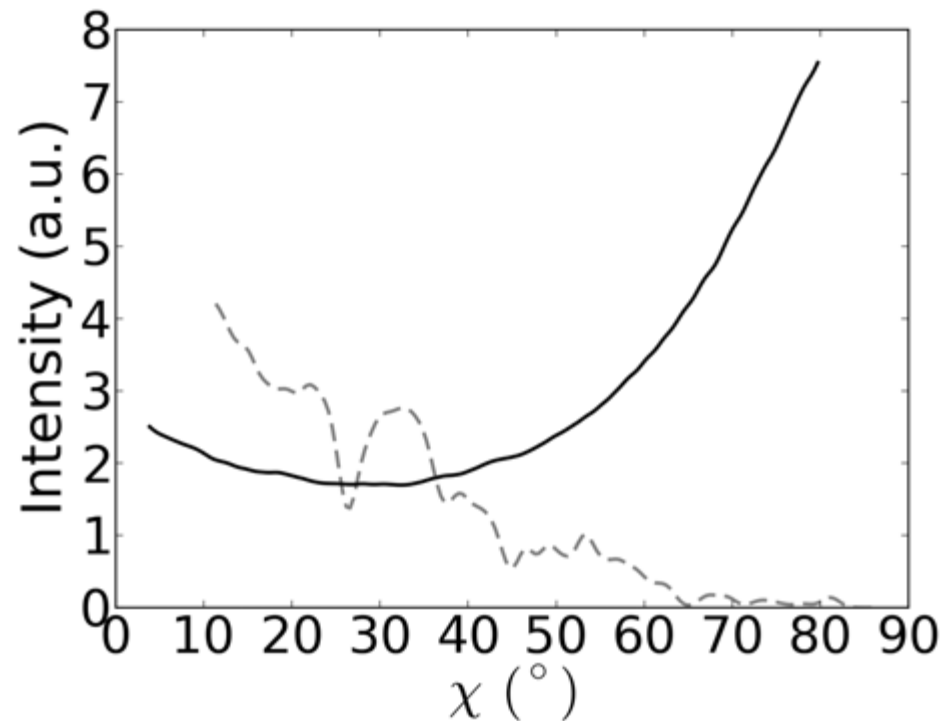
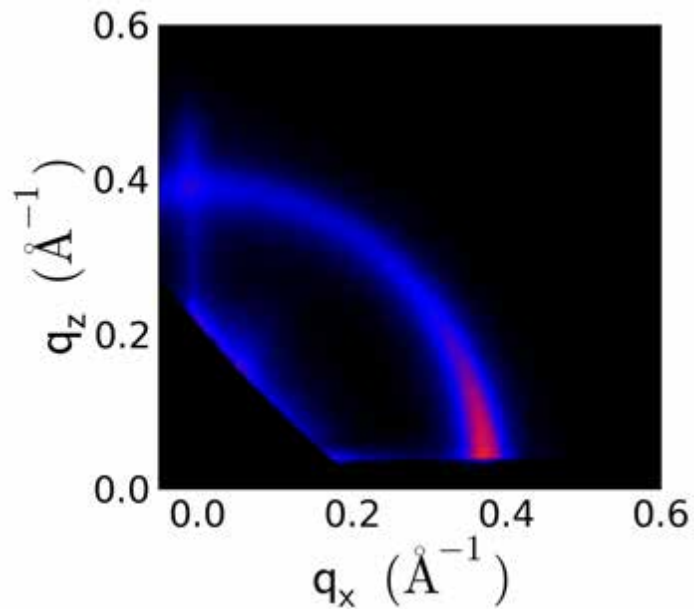
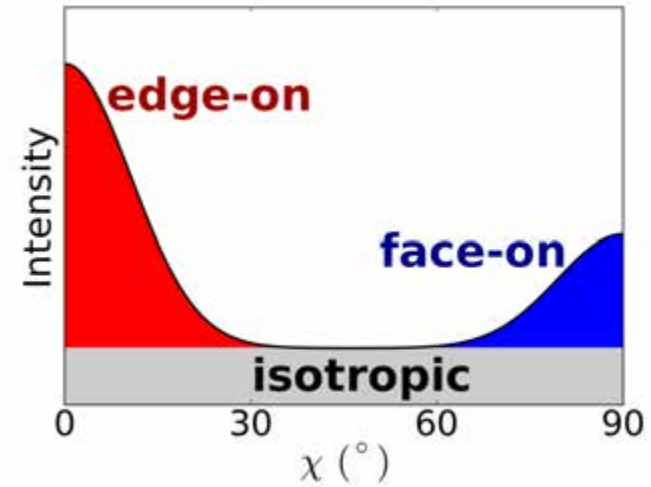
What about in-plane alignment?

- Chains align with the grooves
- Not an in-plane powder!



In-plane aligned

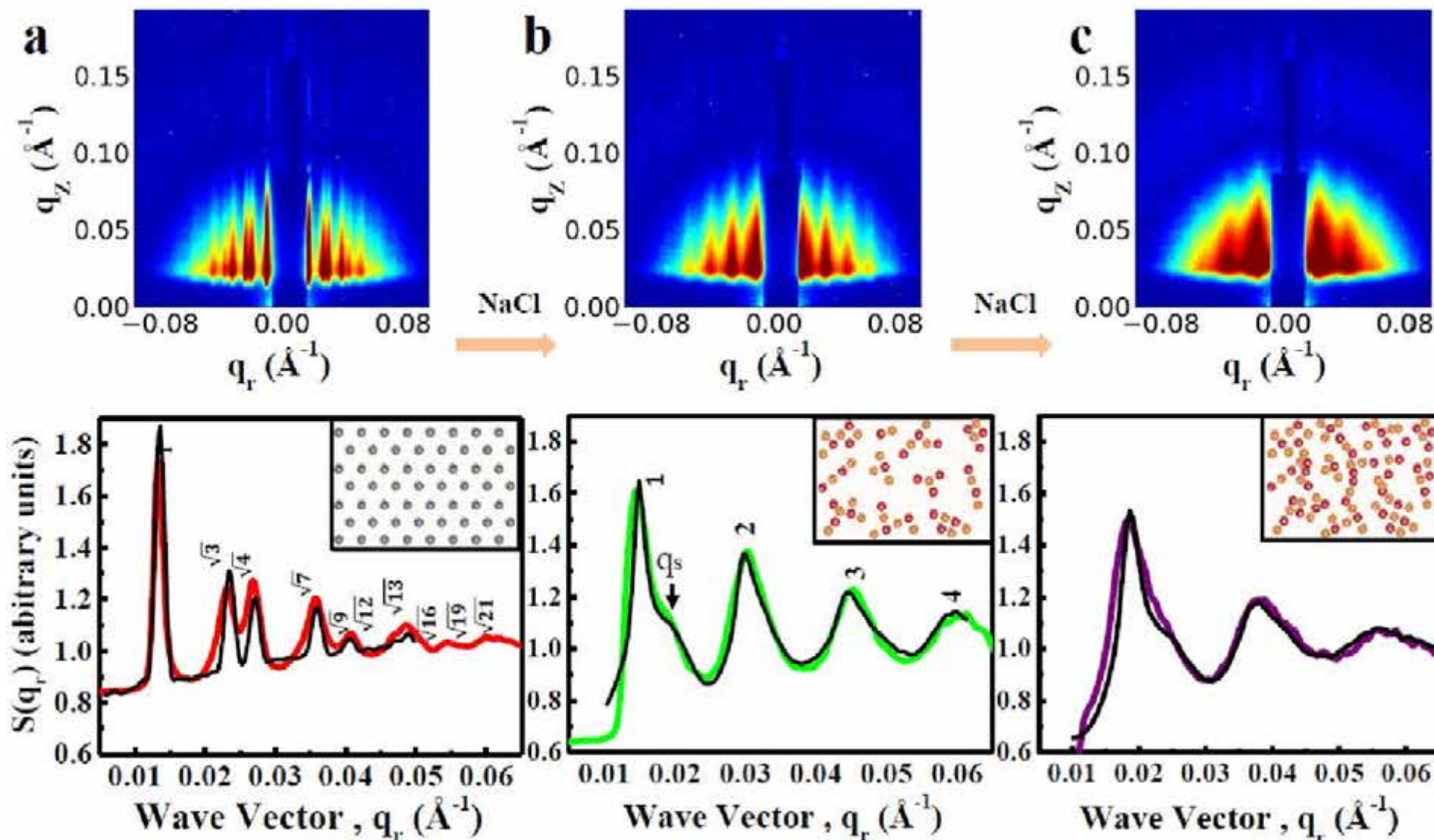
- Rotate sample, accumulated images to reconstruct 3D reciprocal-space
- Use ϕ rocking scan (and correct intensities)
- If very well-aligned: just use intensity at $\phi = 0^\circ$



2D Nanoparticle Assembly

- Nanoparticles attracted to air-water (due to charge)
- Nanoparticle organization controlled by DNA coronas

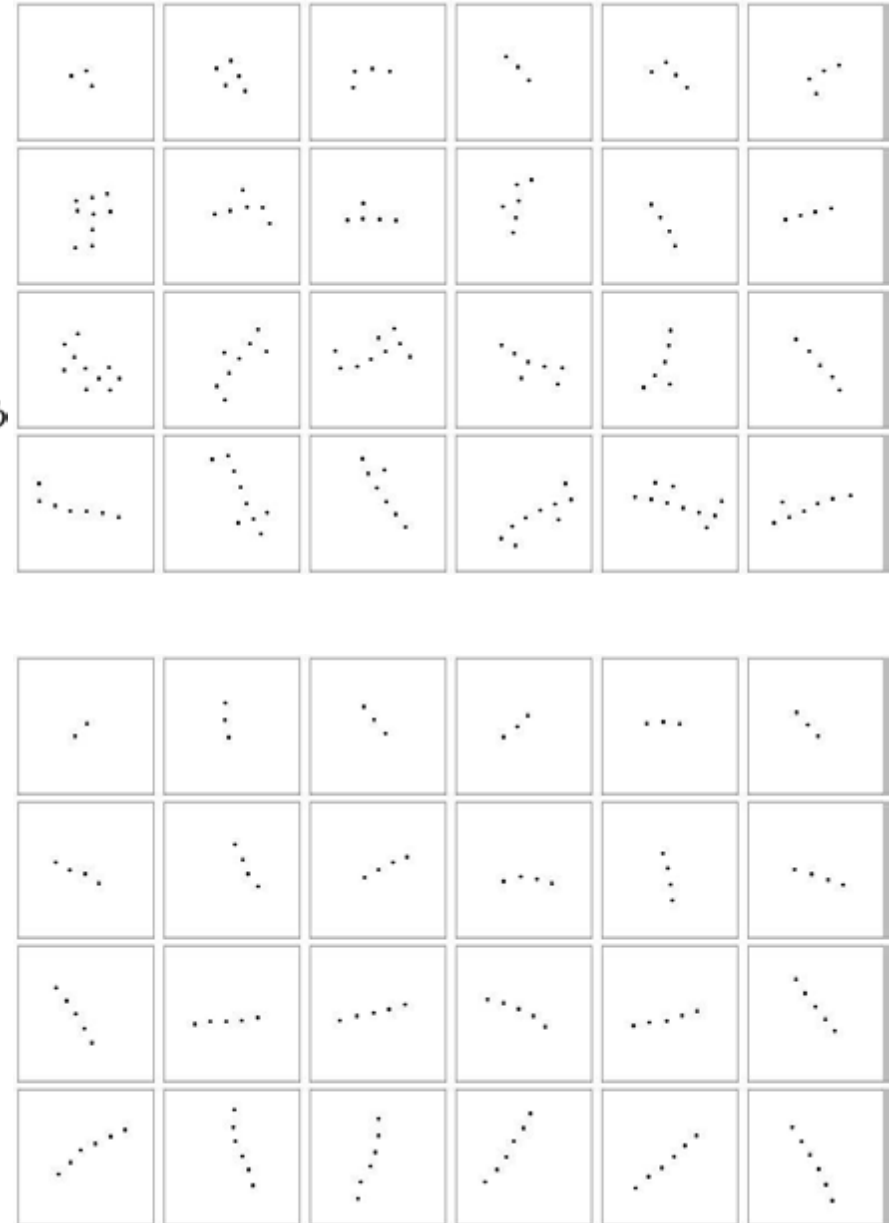
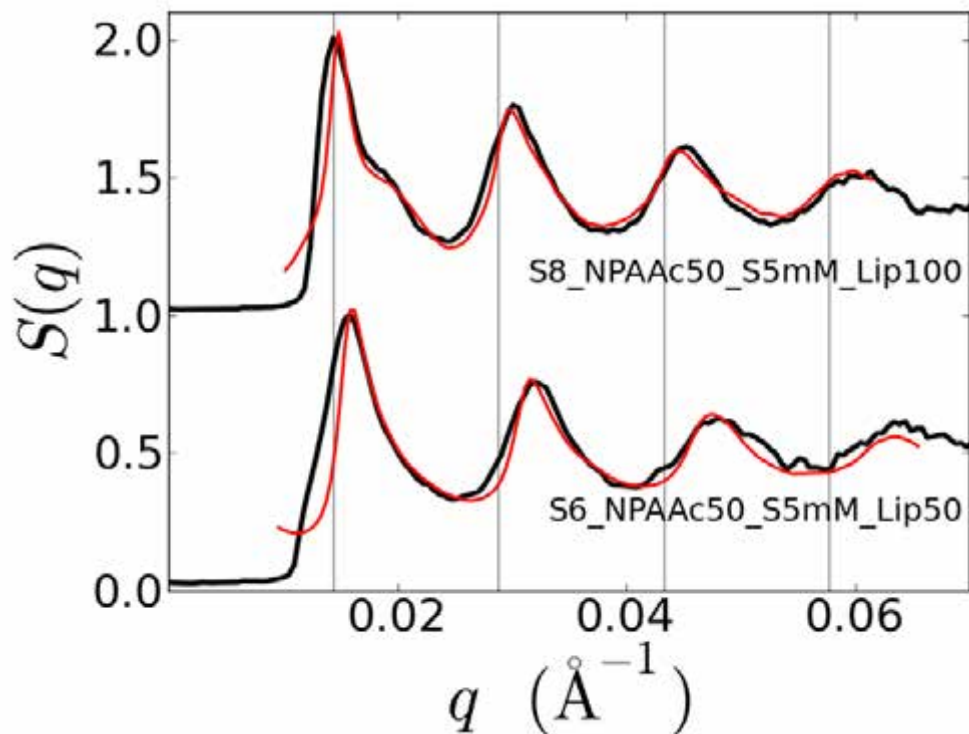
Srivastava *JACS* 2014, ASAP



2D Nanoparticle Assembly

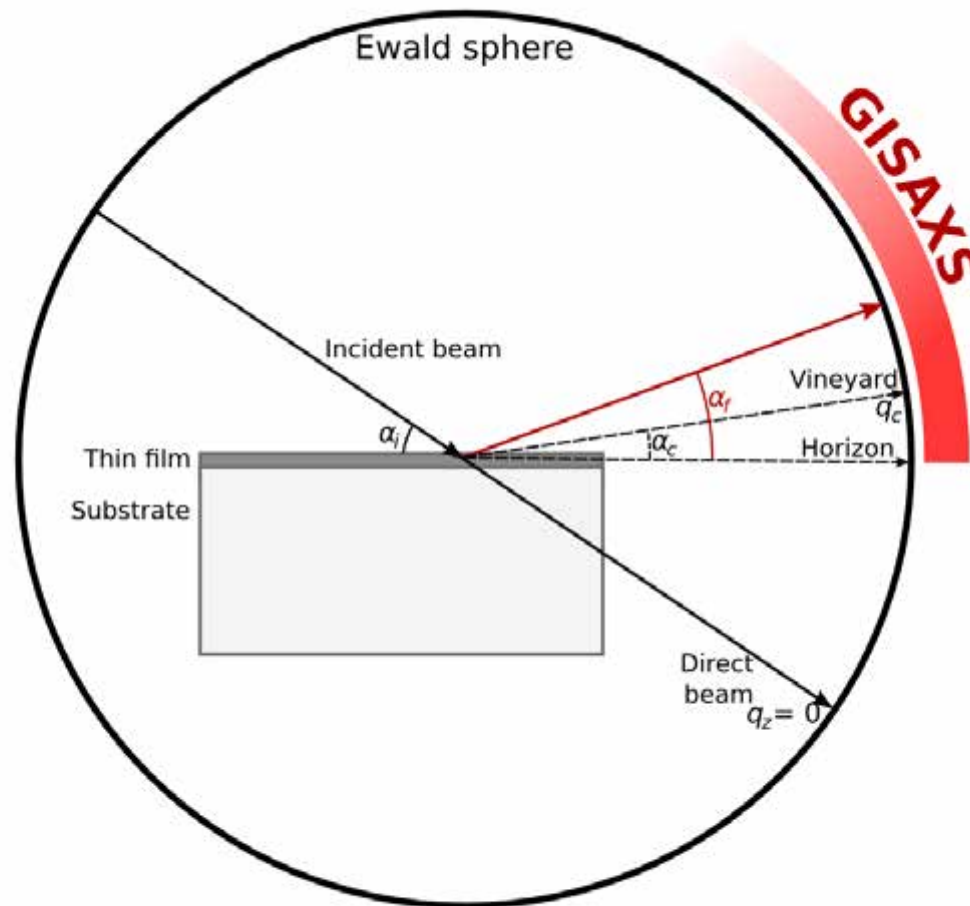
- Brute-force modeling...

$$\begin{aligned} P(q_r) &= \int_0^{2\pi} |F(q_x, q_y)|^2 d\phi \\ &= \int_0^{2\pi} |\rho(r) e^{iq \cdot r} dV|^2 d\phi \\ &= \int_0^{2\pi} \left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho(x, y) e^{iq_x x} e^{iq_y y} dx dy \right|^2 d\phi \\ &= \int_0^{2\pi} \left| \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \rho(x, y) e^{iq_r \sin(\phi)x} e^{iq_r \cos(\phi)y} dx dy \right|^2 d\phi \end{aligned}$$



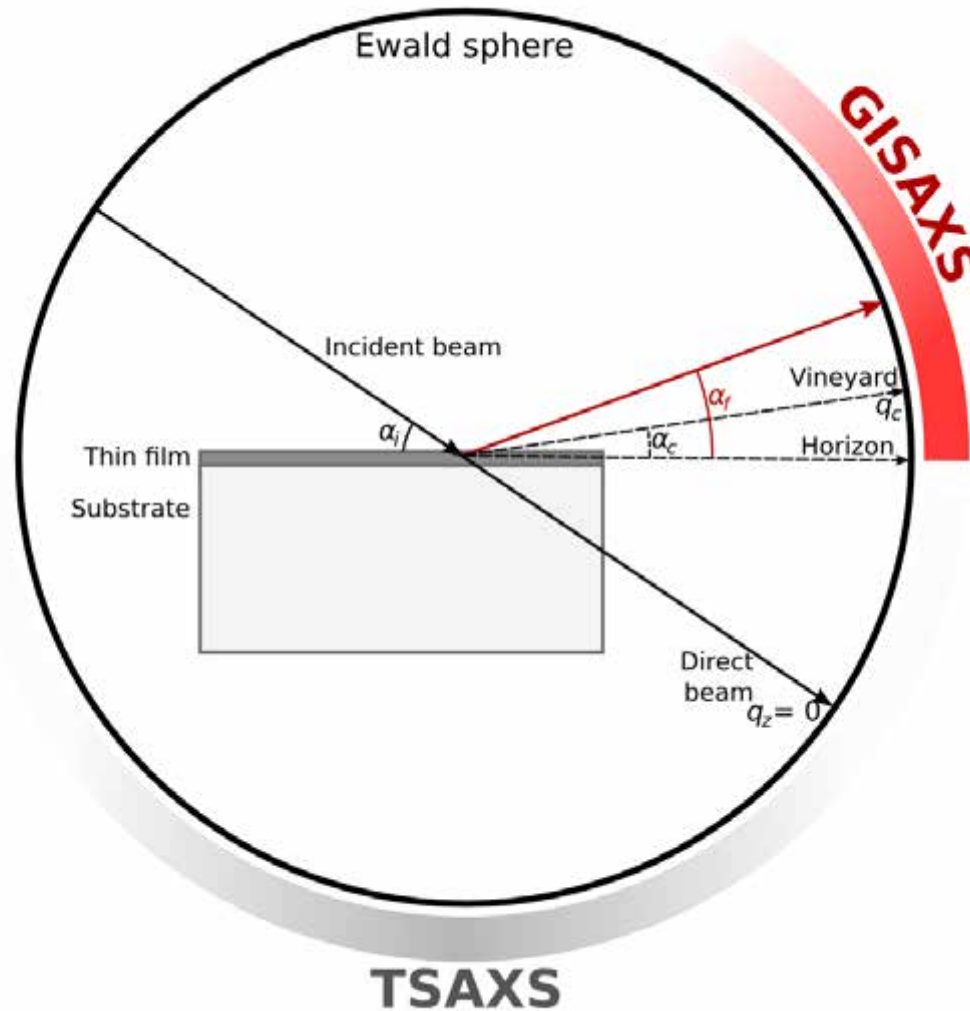
Conventional GISAXS

- Thin film scattering: GISAXS



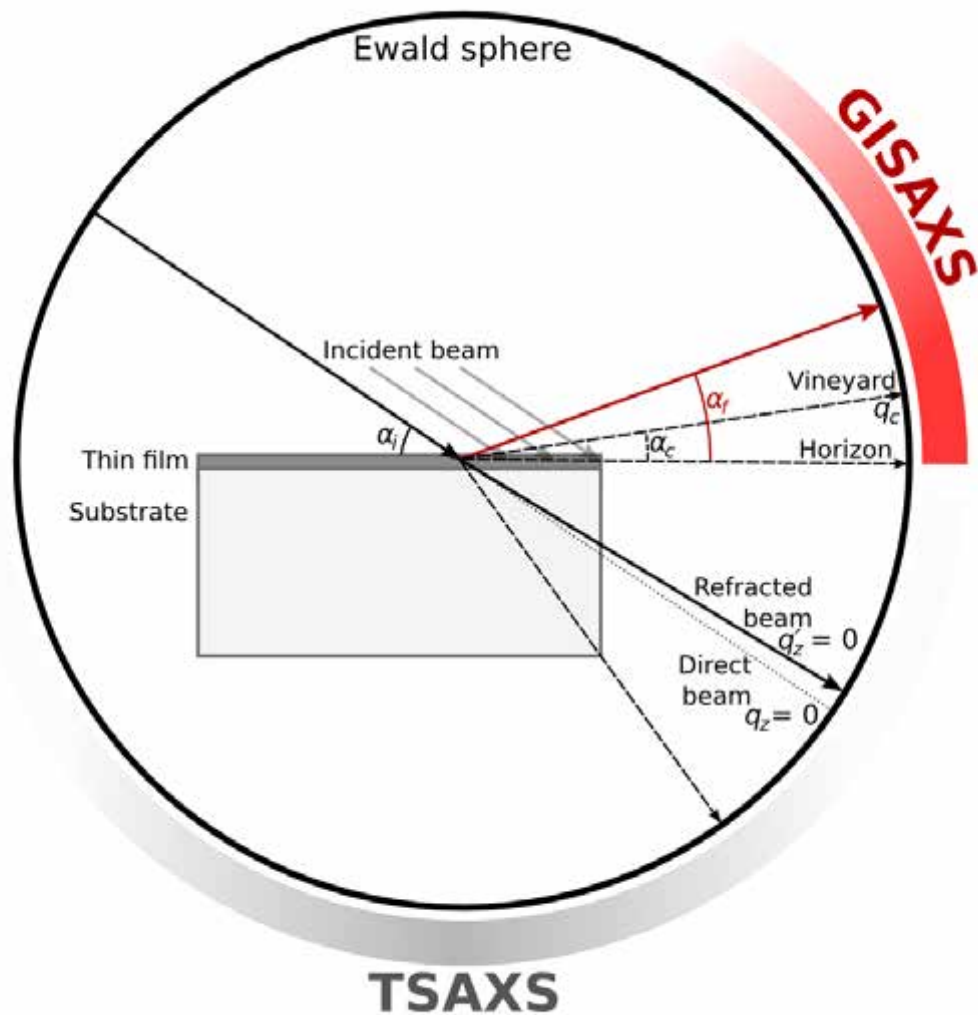
Conventional GISAXS

- GISAXS blocked below horizon
- TSAXS attenuated at high angle



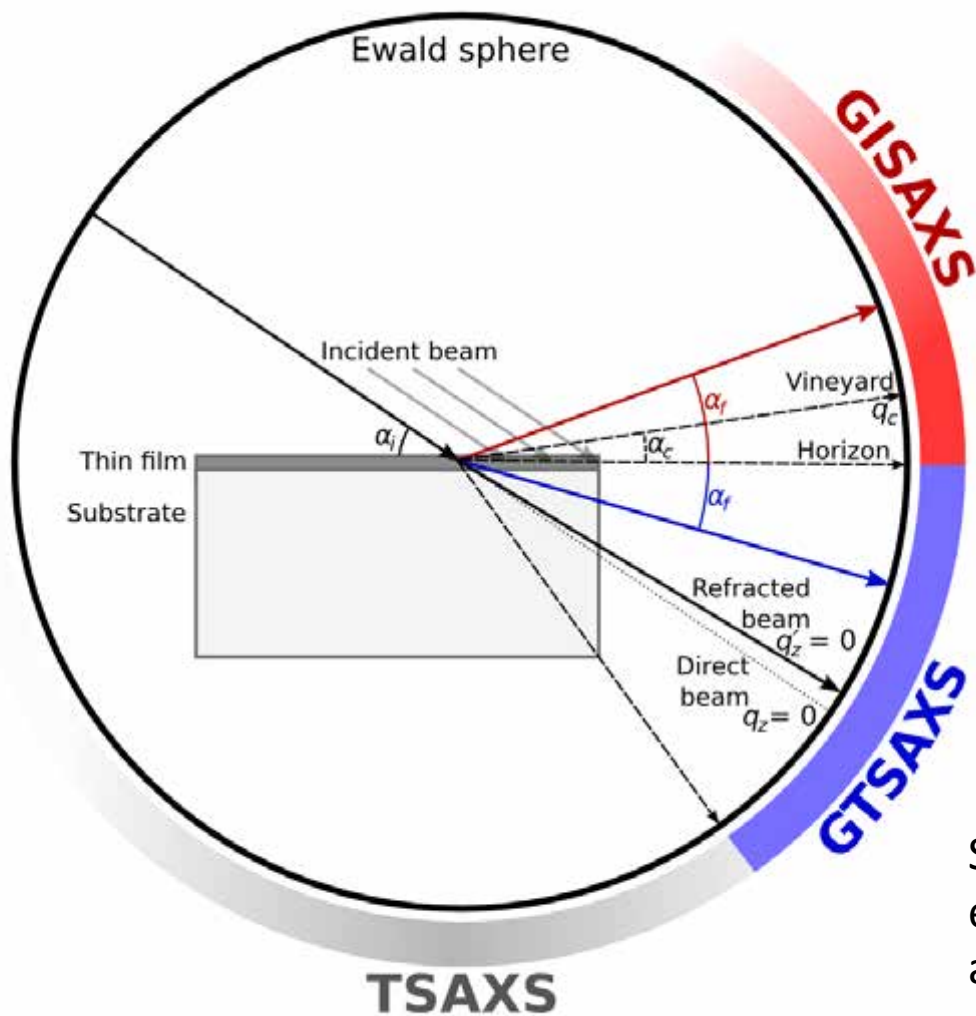
New geometry

- What if beam hits near edge of substrate?
- Scattering exits from edge...



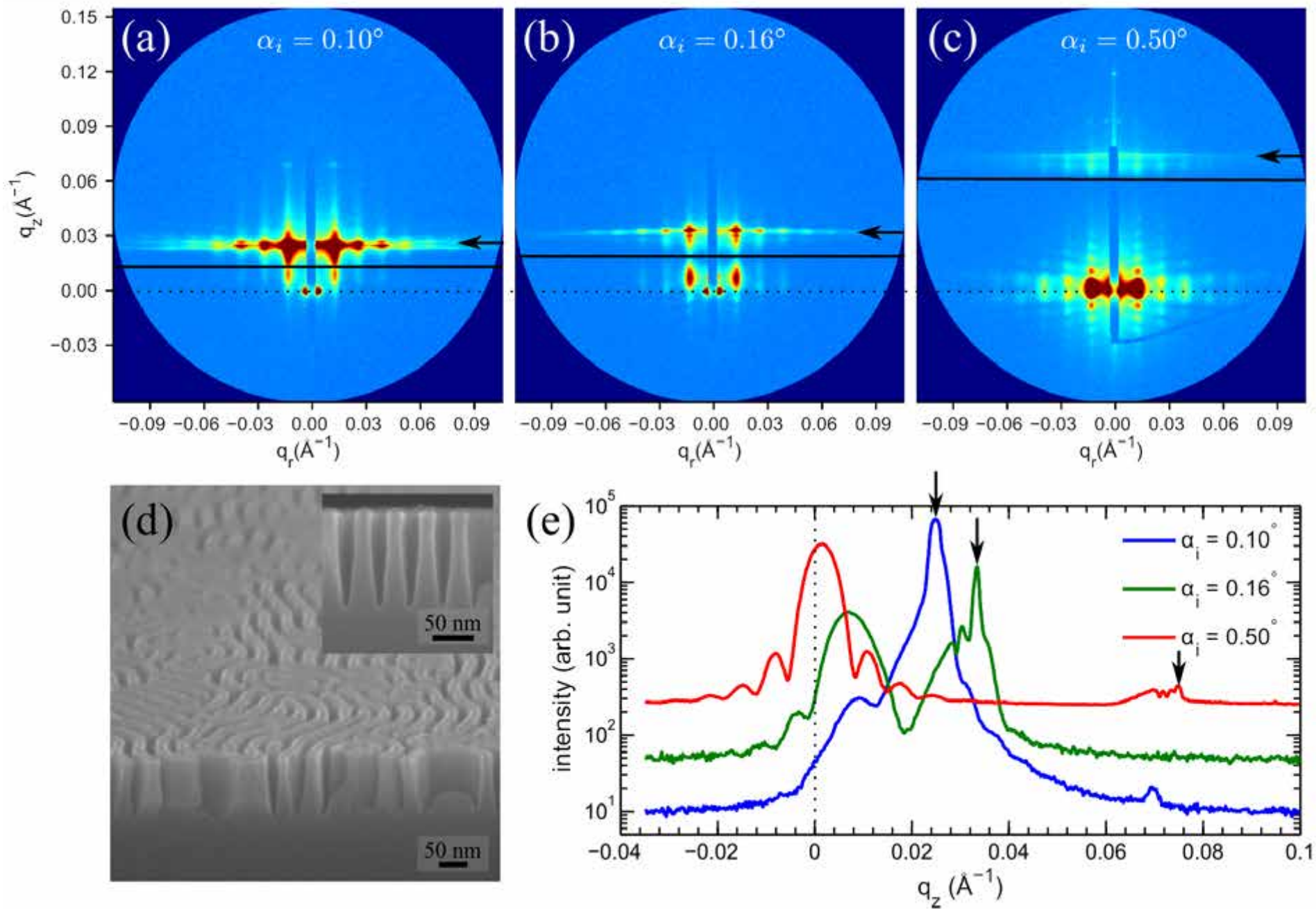
New concept: GTSAXS

- Grazing-incidence Transmission Small-Angle X-ray Scattering



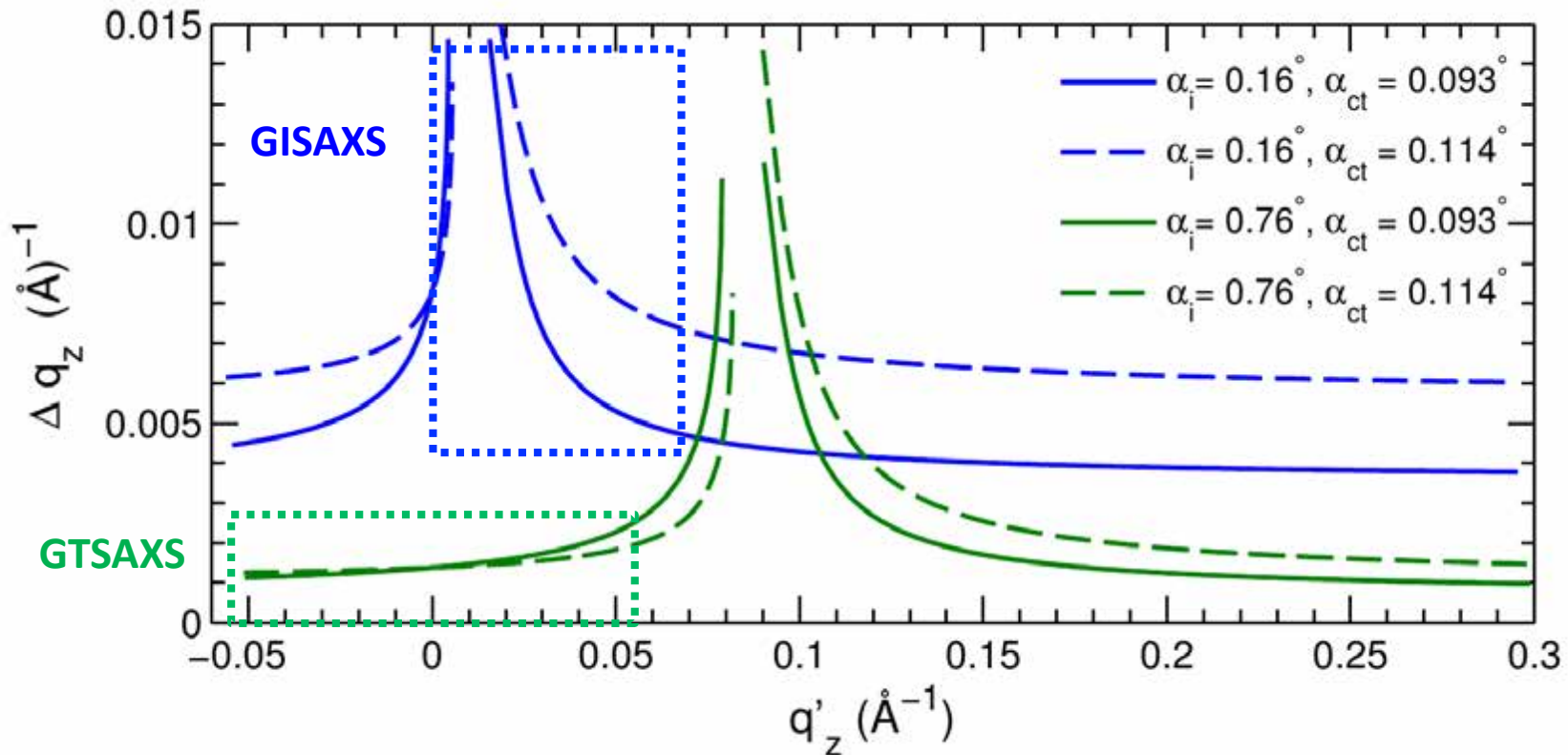
Scattering exits from edge (between horizon and sample corner)

BCP pattern



GTSAXS

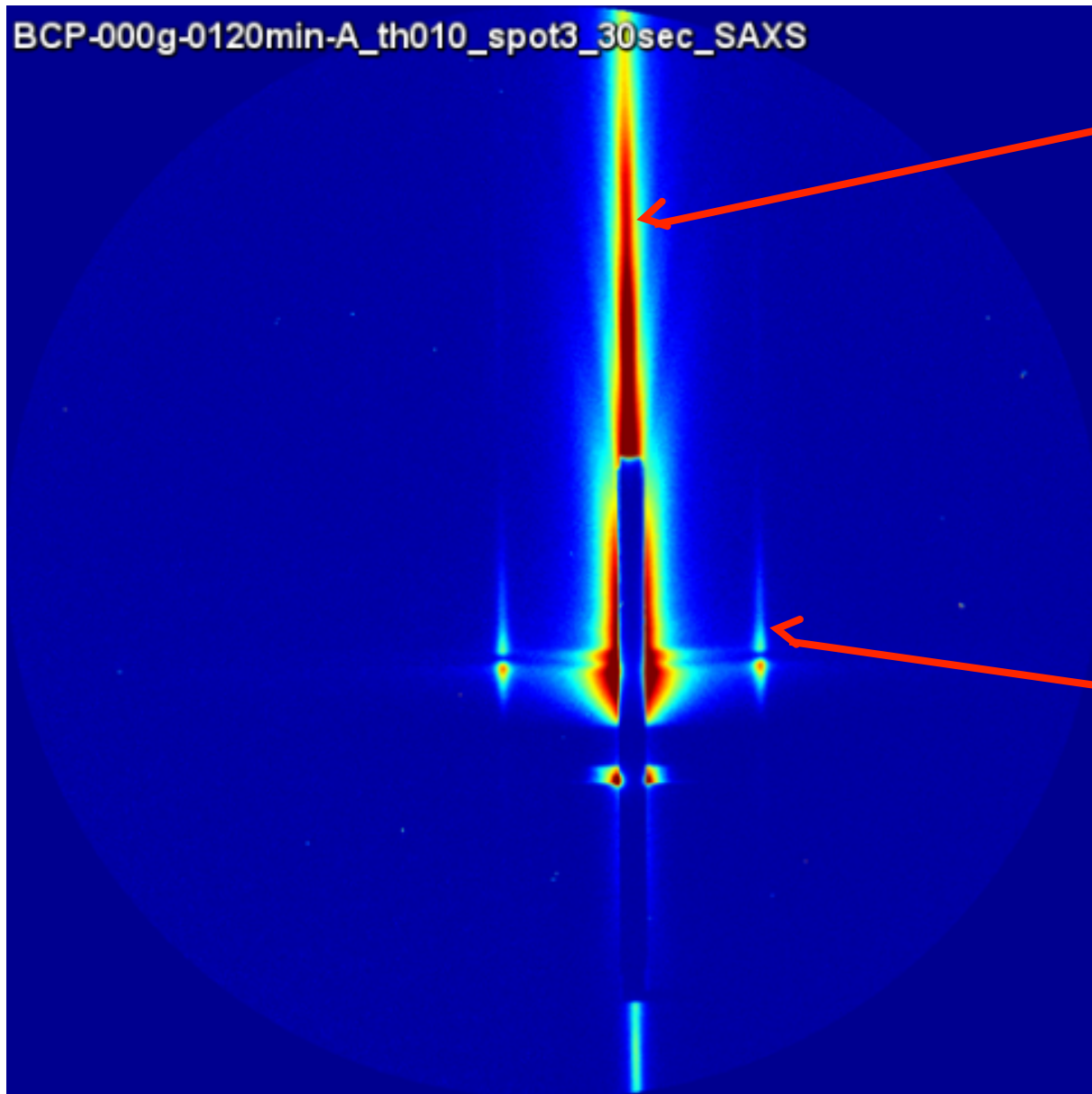
- New geometry makes data much simpler to analyze:
 - Reflection terms nearly zero
 - Refraction is **small** and **linear** (and independent of angle)
- Simple scattering theory (Born approximation) now sufficient to fit data!
- All that is required is (*representative*) sample near edge



Conclusion

- Analysing GISAXS data can be tricky...
- But it's worth doing:
 - In principle, everything about the sample's structure is encoded in the data
- Thinking in terms of **reciprocal-space** can help
- **Questions?**

Exercises

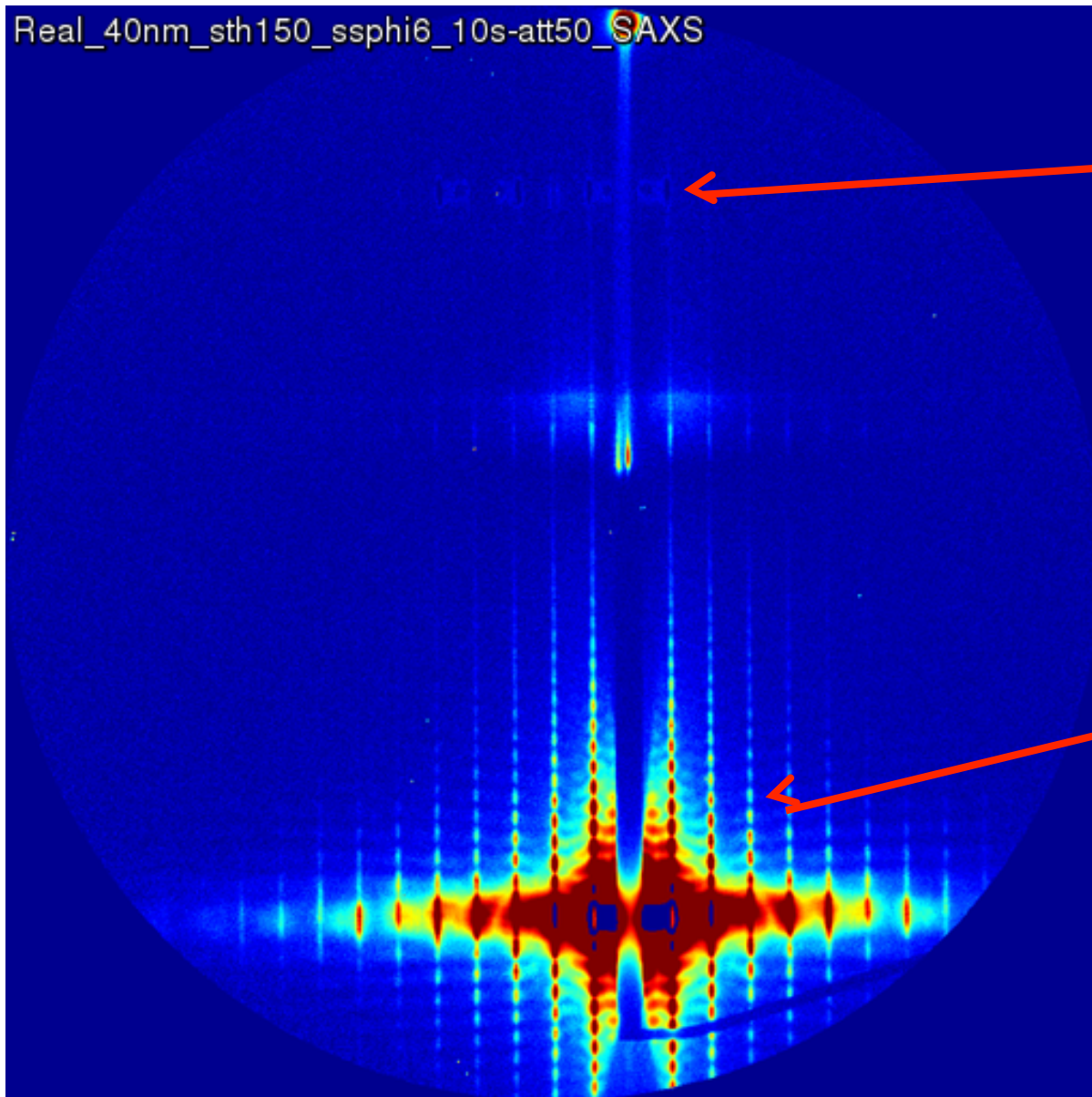


What's this intense rod?

Why are there oscillations along the rod?

What does this peak tell you?

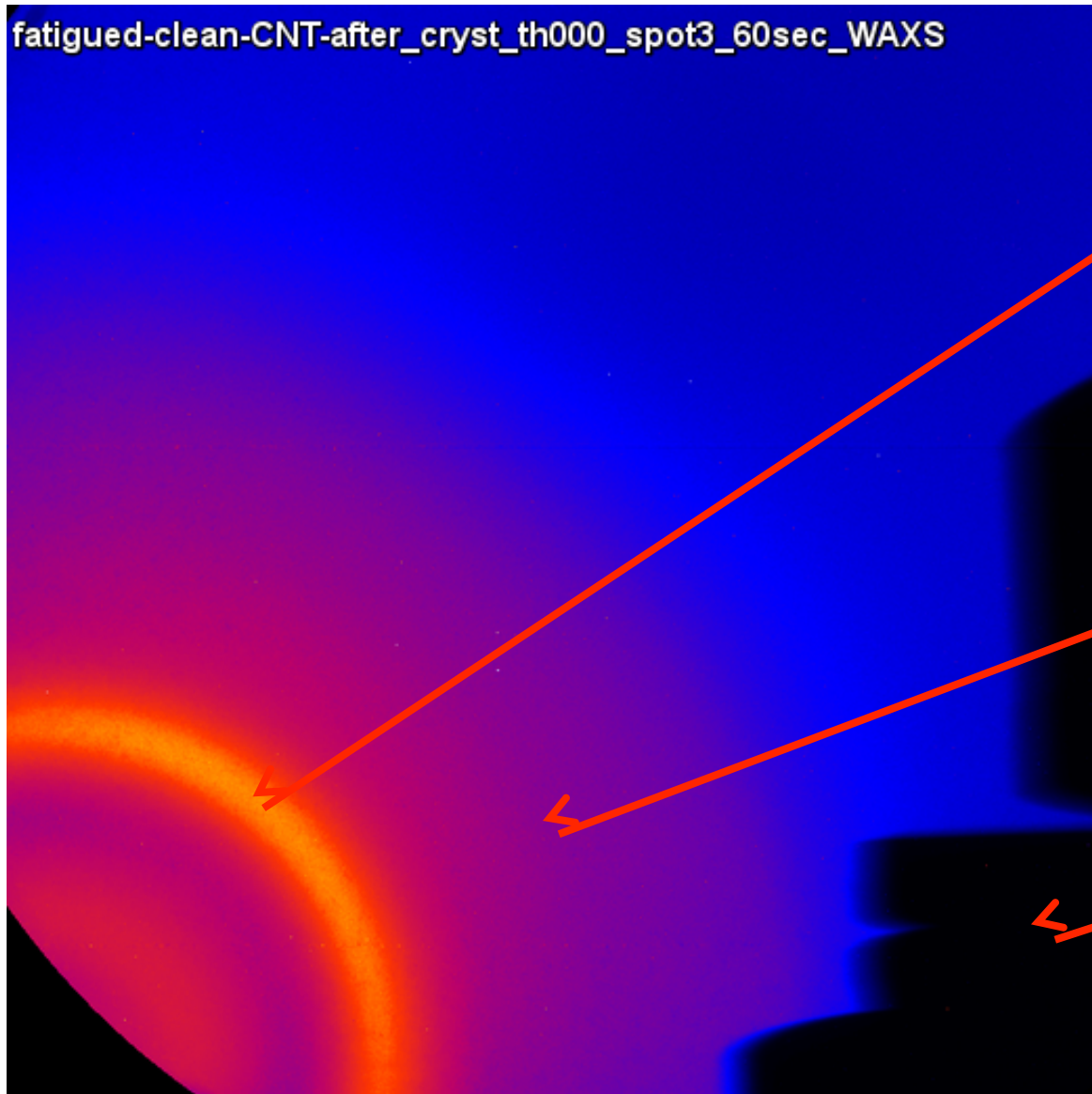
Exercises



What's this stuff?

Lots of peaks!
What kind of sample is this?

Exercises

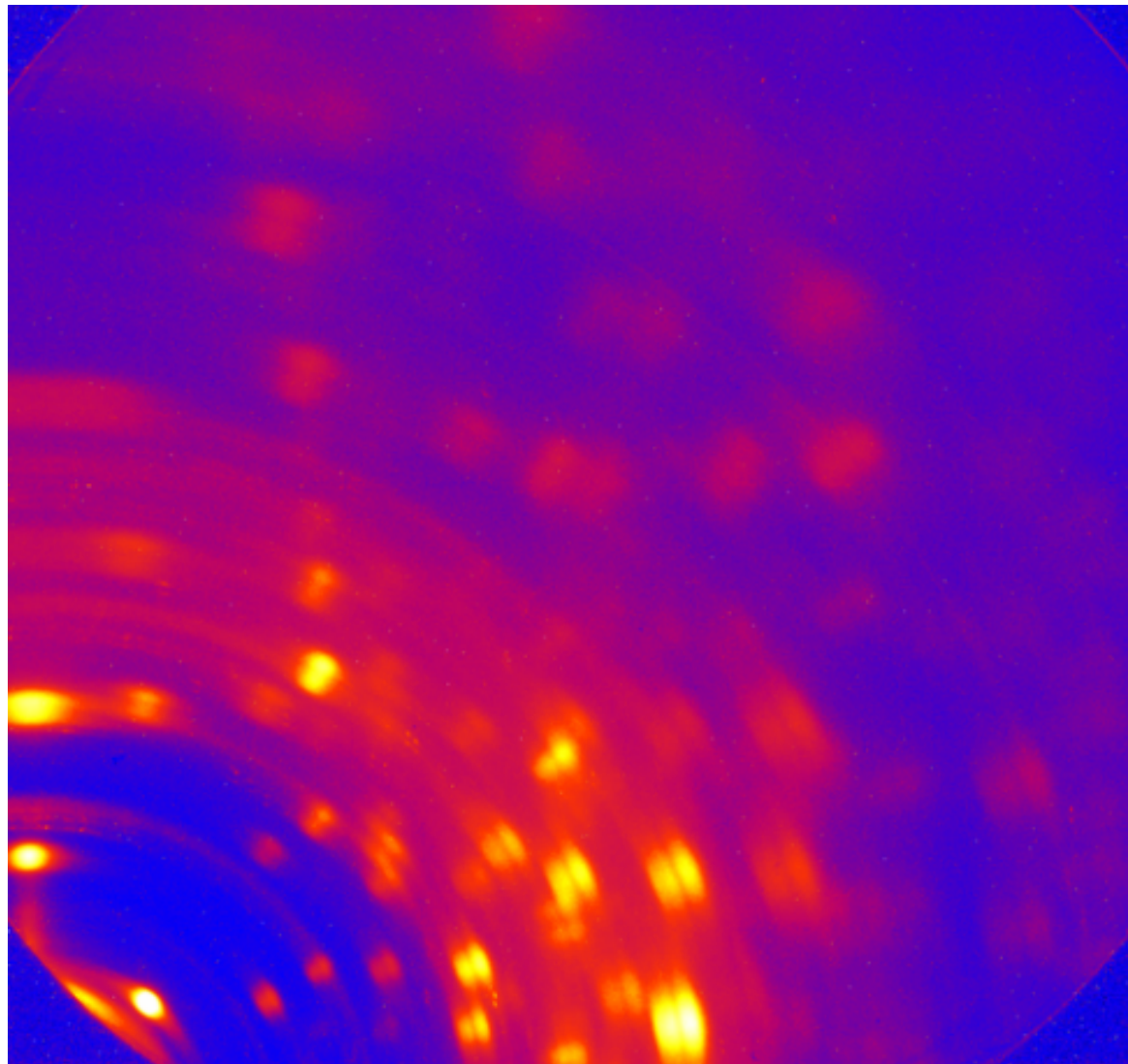


What does this ring tell you?

High background... why?

What's this?

Exercises



How would
you analyze
this sample?